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Part I

AN APPLICATION OF FRACTURE CONCEPTS TO THE PREDICTION OF CRITICAL LENGTH OF FATIGUE CRACKS

Part I. A Review of Pertinent Aspects of Fracture— (Development of Relevant Concepts of Linear Elastic Fracture Mechanics)

SIDNEY O. DAVIS

TECHNICAL REPORT AFML-TR-70-202, PART I

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NOTICE

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FOREWORD

This report was prepared by the Processing and Nondestructive Testing Branch (LLN), Metals and Ceramics Division, Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio. The work was initiated under Project No. 7351, "Metallic Materials," Task No. 735108, "Processing of Metals." The research was conducted in the Air Force Materials Laboratory by Mr. Sidney O. Davis (LLN), Project Engineer.

This report, published as Parts I, II, III, IV, and V, covers work accomplished from June 1969 to June 1970 and was submitted by Mr. Davis in partial fulfillment of the requirements for a Master's Degree in Metallurgical Engineering at The Ohio State University, Columbus, Ohio. It also provides the Government and industry with technical information required in the application of fracture mechanics concepts. This report was submitted by the author in August 1970.

The author acknowledges and expresses his appreciation to the Air Force Materials Laboratory, Air Force Systems Command, under the directorship of Dr. Alan M. Lovelace, for the sponsorship of the full time training to obtain the Master's Degree in Metallurgical Engineering with emphasis on the Mechanical Metallurgy discipline. Appreciation is also expressed for the use of the materials, fabrication facilities, test specimens, laboratory facilities, and test equipment.

The thesis research work was accomplished under the direction of Professor J. W. Spretnak, Department of Metallurgical Engineering, The Ohio State University, internationally prominent in the field of Mechanical Metallurgy. The author expresses his appreciation to Dr. Spretnak for his guidance and counselling during the course of this investigation.

The author also acknowledges the cooperation and support of Dr. Harris M. Burte, Chief of Metals and Ceramics Division; Mr. T. D. Cooper, Chief of the Metallurgical Processing and NDT Branch; and Mr. V. DePierre, Technical Manager of Metallurgical Engineering.

This technical report has been reviewed and is approved.

T. D. Cooper

T. D. COOPER
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ABSTRACT

The purpose of this report is to synthesize technological concepts of fracture by making a historical review of the literature from 1913 up to the present time. The pertinent aspects of fracture and the development of relevant concepts of linear elastic fracture mechanics derivatives were delineated and summarized for the prediction of the critical length of fatigue cracks. The pertinent aspects of fracture consisted of the synthesis of Ingliss, Griffith, Orowan, Irwin, and Westergaard's relevant theoretical concepts. It also delineates Boyle's analytical and experimental results of the Westergaard-Irwin theoretical compliance of through-the-thickness centrally cracked plate and sheet for the determination of plane-strain (K_{Ic}) and plane-stress (K_c) fracture toughness stress-intensity parameter of high strength alloys.

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SECTION I

INTRODUCTION AND BACKGROUND

The proliferous performance requirements for aeronautical and aerospace missile systems have resulted in needs for higher strength structural alloys (Reference 1). The specified requirements of the minimum possible weight in metallic structural components of aircrafts and missiles' rocket motor casings have led to the escalation of the use of very high strength steel, aluminum, and titanium alloys (Schematic A of the Appendix). Examples are steels with yield strengths greater than 200,000 PSI and both ferrous and nonferrous alloys having a strength-to-density ratio greater than or approximately equal to 700,000 PSI/lb per cubic inch or inches. At these strength levels, many structural engineering alloys have exhibited brittleness in the presence of small flaws under a gross uniaxial state of an externally applied load. These flaws may be presented in the alloy as a result of casting and/or shaping and forming processing. They could have been induced during the fabrication of thin or thick-wall structural components or pressure vessels. In addition cracks or flaws may have developed in the alloy from obvious stress raisers (stress concentrators) during in-service applications (Reference 2). As a result, service failures in high strength alloys often occur by cataclysmic fracture (brittle failure) rather than by gross yielding and distortion. As a result two design criteria have evolved as shown in Schematic B of the Appendix. In addition, these brittle failures happen at stress levels well below the macro-elastic design stresses, which are usually based on the engineering 0.2 percent offset yield strength of the material (Reference 3).

The complete elimination of minute surface defects and ultimately fatigue cracks in airframe and missile rocket casings structural alloys is not within the practical framework of the present state-of-the-art of metallurgical technology. Therefore, the "fail safe" design concept has been developed to provide adequate structural integrity and safety in the presence of such conditions of metallic materials damage that is exposed to external loads. According to this concept, multiple load paths or crack arrestors are designed into the structure so that a propagating crack cannot destroy the load-carrying ability of the entire structure (Reference 4). However, even in commercial

aircraft, it is not feasible to design all load carrying members on a "fail safe" basis. For an example, the landing gear of aircraft are usually single load path structures. This is made necessary by weight and space considerations. As a consequence, the need for exceptionally reliable aircraft and missile casing metallic structures has brought to a critical state the dire need for an intrinsic material parameter that will measure the resistance to cracking of a high strength alloy for the prevention of catastrophic failure under tensile (static) and fatigue (dynamic) loading conditions in service. The "fracture toughness" of a high strength metal or alloy has become of prime importance as the fracture resistance parameter that appears to meet the required measurable criteria. This parameter is believed to be an intrinsic property of the material. Therefore, if so, it should ideally measure the alloy's ability to dissipate increasingly stored elastic strain energy as localized plastic flow at the tips of defects or cracks under externally applied tensile loads. This ideal transfer of energy would also appear to reduce the possibility of the slow growth and rapid propagation of cracks at stress levels below the yield strength of alloys under external loading conditions.

It has been demonstrated (Reference 3) that the fracture toughness of high strength alloys is one of the major determinants of the thickness and inherent stress level that an alloy can be reliably used in the design of highly tensile stressed structures fabricated of high strength alloys. The degree of toughness of an alloy has been among the most important properties contributing to the success or failure of specific aircrafts and missiles (Reference 5). A knowledge of the fracture toughness, fatigue crack propagation, corrosion resistance, and the strength remaining at any stage of development of a crack is important in the application of the fail safe concept to structural components. The need for this knowledge becomes more acute as vehicle performance requirements increase at the expense of decrease material performance and reliability. As the exploration of space continues, the consequences of alloy structural failure, as in manned spacecraft, become continually more serious.

The purposes of this study (Parts I, II and III) were to synthesize technological concepts of fracture by making a historical review of the literature and to determine (Parts IV and V) experimentally if high strength alloys fracture mechanics concepts could be applied to the prediction of fatigue crack's slow

growth and the onset of rapid propagation under plane-strain conditions at the tip of a crack. The primary aim was to add to the knowledge of fatigue crack propagation technology in high strength alloys selection and utilization in design. The mode of failure under which the fracture toughness parameter of an alloy is deduced from fracture mechanics concepts was compared with the failure under fatigue crack propagation (Part V) to establish whether the use of the fracture concepts were technically valid in predicting critical crack lengths at the onset of rapid propagation under fatigue loading conditions. The degree of correlation, quantitatively, between the fracture mechanics concepts and fatigue crack propagation predictions was pursued (Part V) to establish whether the critical crack length at onset of catastrophic failure under fatigue conditions could be predicted from fracture toughness experimentally obtained data and fracture mechanics mathematical concepts deduced in Part IV.

The mode of crack propagation failure under uniaxial tensile loading (static) to obtain fracture toughness experimental data compared to the mode of failure under uniaxial fatigue loading (dynamic) were investigated in Part V relative to the critical crack length determination at onset of cataclysmic fracturing.

In the past, it has been deduced by investigators, from a pure mechanics point of view, that fracture toughness properties and concepts of high strength alloys can be used to predict critical crack lengths under fatigue crack propagation conditions without rigorous experimental verification rationalized by the discipline of mechanical metallurgy (mechanics plus physical metallurgy). As a consequence, there was a need for a more intensive evaluation and analysis from a mechanical metallurgy rationale.

The analytical and experimental studies of fracture toughness K_{Ic} parameter ability to predict the critical length of fatigue cracks and the observations of crack propagation velocities were conducted on 7075-T7351 aluminum alloy, as shown in Parts IV and V.

Cracks were experimentally propagated in aluminum plate specimens under static (tensile strain rate of .05 inches/min.) and dynamic conditions (conventional fatigue cyclic rates of 2000 cycles/min.). Stress-intensity factors at the tips of through-the-thickness fatigue cracks and critical crack lengths experimental data were obtained and analyzed, as shown in Part V.

SECTION II

THE PROBLEM

1. STATEMENT OF THE PROBLEM

The purposes of this project were to:

a. Predict the critical length of fatigue cracks under the stable fracturing mode and to experimentally substantiate the analytical prediction (Parts IV and V).

b. Study the critical crack length at instability under fatigue conditions in plate specimens under plane-strain (depicted in Part V).

2. SIGNIFICANCE OF THE STUDY

a. A chronological and composite historical review of the: (1) development of concepts of linear elastic fracture mechanics (Part I), (2) theoretical and analytical aspects of fatigue (Part II), and (3) the thermodynamics of fracture (Part III) was documented.

b. The quasi-static plane-strain (K_{Ic}) fracture toughness of 7075-T7351 aluminum alloy was determined experimentally on 4-, 5- and 6-inch wide plate specimens and compared with data in the literature (Parts IV and V).

c. The behavior of the plastic zone under quasi-static and stable crack propagation in fatigue was studied in Part V by:

(1) observing the size of the plastic zone under quasi-static and fatigue testing conditions

(2) observing whether there was stable extension under static loading conditions

(3) determining experimentally the stable and unstable crack growth velocities under fatigue conditions

(4) determining experimentally under stable and unstable crack propagation whether the process was isothermal or adiabatic using liquid crystals

(5) determining the effectiveness of variable stress concentration factors at the tip of a fatigue crack

(6) comparing the predicted critical crack length in fatigue conditions with that predicted from quasi-static fracture toughness parameters.

d. The author has compiled a relatively complete bibliography on fracture.

e. Although all references listed in the bibliography are not used in the text of this report, they are listed for future use by those individuals who are working on the details of fracture in research or technology.

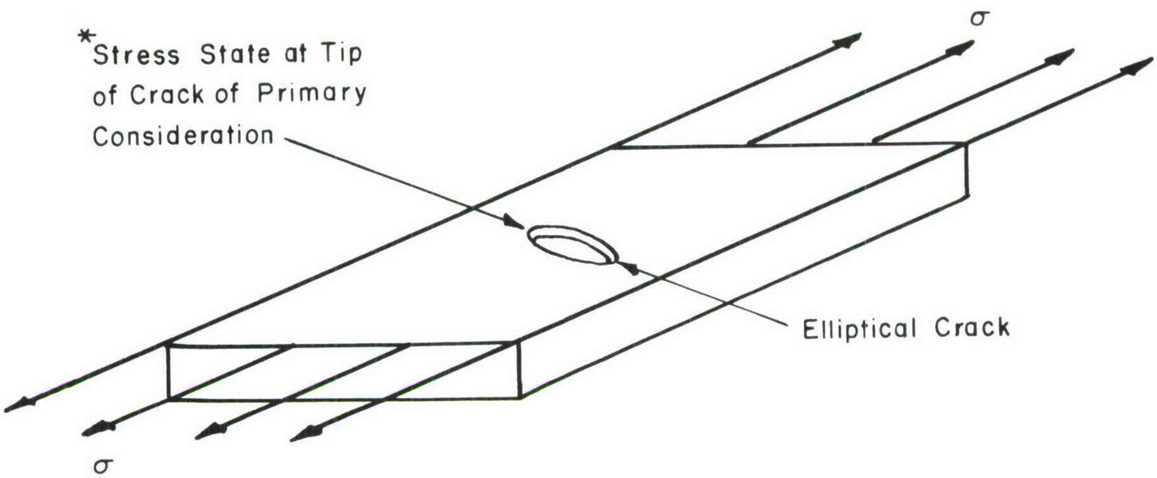
SECTION III

REVIEW OF PERTINENT ASPECTS OF FRACTURE

1. DEVELOPMENT OF RELEVANT CONCEPTS OF LINEAR ELASTIC FRACTURE MECHANICS

a. Continuum Fracture Mechanics Concepts

(1) Ingliss-Griffith Basic Theoretical Crack Model and Concepts for an Elastic Solid (Glass)

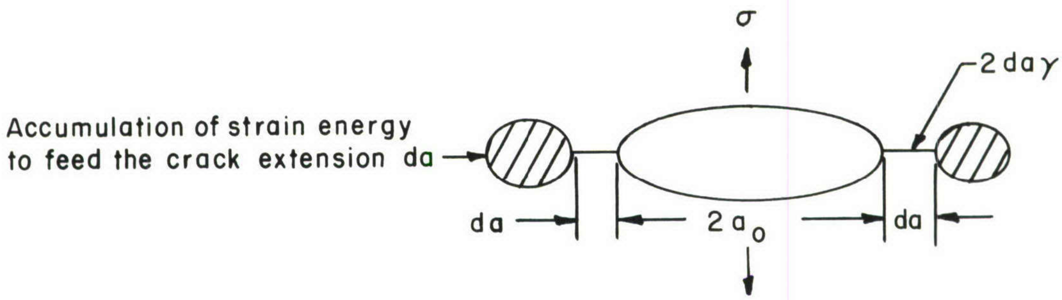


*Plane-Strain Conditions consist of tensile strains in two directions and stress in three directions with respect to the X-Y-Z Cartesian coordinates.

*Plane-Stress Conditions consist of tensile strains in three directions and stress in two directions with respect to the X-Y-Z Cartesian coordinates.

Figure 1. Griffith Model: Perfectly Elastic Solid (Reference 6)

Griffith used the elliptical crack model, because in 1913 Ingliss presented the stress analysis solution for elliptical cracks embedded in an elastic solid. The elliptical crack model details are as follows (References 7 and 8):



Without the crack in the elastic solid, the strain energy density is:

$$\frac{\sigma^2}{2E} \tag{1}$$

where:

σ = the stress = the intensity of a force reaction within the elastic body on the application of an external uniaxially applied load.

E = the modulus of elasticity of the solid body.

Ingliss' solution gives the accumulated energy density at the tip of the crack as:

$$\delta_E = \frac{\pi a_0^2 \sigma^2}{E} \tag{2}$$

where a_o = 1/2 the original crack length before the accumulation of strain energy required to feed the crack extension da. The corresponding surface energy associated with the crack which was created as a result of the transformation of strain energy to surface energy is:

$$4a\gamma \tag{3}$$

where γ = the unit surface energy

Physical Significance of γ:

(1) Surface free energy $= \gamma \equiv \left(\frac{\partial F}{\partial A}\right) T, V = f(A, V, T)$

where:

∂F = change in free energy (ergs) as a function of temperature (T) and volume (V).

∂A = change in free surface area (cm^2) as a function of temperature (T) and volume (V).

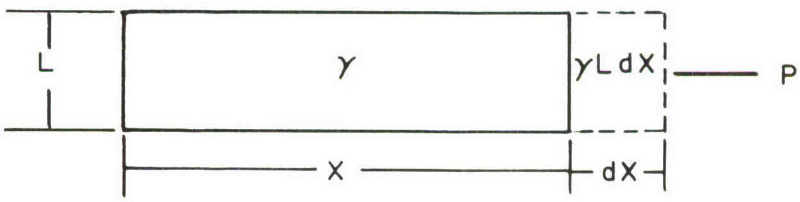
(2) At high temperature and for small systems, the system will attempt to reach minimum "surface free energy" in terms of minimum area per unit volume $\frac{\partial A}{\partial V}$, a sphere

(3) This means that a fine wire will tend to contract

(4) Applied load (P) tends to extend the wire

(5) At zero change in length (X), the applied load (P) just balances the "surface energy" force. This point of equilibrium of the surface is the experimental method of determining the "surface free energy" γ ($ergs/cm^2$).

Analogy Between Surface Energy (γ $ergs/cm^2$) and Surface Tension ($\frac{P}{L}$ dynes/cm):



Soap Bubble Model

therefore:

$$dE = \gamma L\, dX = P dX$$

where:

dE = Change in total surface free energy

therefore:

$$\gamma = \frac{P}{L}$$

where:

γ = surface energy in ergs/cm^2

$\frac{P}{L}$ = the surface tension in dynes/cm

The criterion for crack extension is that the decrease in stored strain energy of the elastic solid is greater than the increase in surface energy. As a consequence, the accumulation of the maximum strain energy at the tips of the crack is dissipated as surface energy in terms of creating free surfaces (cracks) in the elastic solid.

The equilibrium length of the crack is:

$$\frac{d}{da} \frac{(4a\gamma - \pi a^2 \sigma^2)}{E} = 0 \tag{4}$$

therefore:

$$4\gamma = \frac{2\pi a \sigma^2}{E} \tag{5}$$

$$a \text{ equilibrium} = \frac{2\gamma E}{\pi \sigma^2} \tag{6}$$

$$\sigma_c^2 = \frac{2E\gamma}{\pi a} \tag{7}$$

where σ_c = the strength of the plate or the critical stress before fracture

$$\therefore \quad a \propto \frac{1}{\sigma^2} \tag{8}$$

The fracture strength of a plate is dependent on the size of the crack and is expressed as:

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a}}$$

σ_c and a_o having approached a condition of a critical are criterion for the conditions of unstable growth of a crack or the onset of unstable crack propagation from a physical significance point of view. Unstable cracking is "catastrophic propagation" of a crack in an elastically stressed body.

In summary, the Basis of Fracture Mechanics is as follows:

$$a \text{ equilibrium} = \frac{2\gamma E}{\pi \sigma^2}$$

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a}} \quad \text{as } a_0 \rightarrow a_0 + da$$

(2) Griffith-Orowan-Irwin Basic Theoretical Concepts as Applied to an Elastic-Plastic Solid (Metals)

From an energy view according to Griffith Theory and Irwin and Orowan criteria, fracture toughness of a material is the component of work irreversibly absorbed in local plastic flow (P) and total surface energy (γ) to create a unit surface area of fracture. For glass and ceramics the general functional tensile stress (σ) to propagate a crack (a) is as follows:

$$\sigma \approx \sqrt{E\gamma/a} \tag{9}$$

where:

γ = Total surface energy of fracture surface a

E = Young's modulus

For metals, the general functional tensile stress (σ) to propagate a crack (a) is as follows:

$$\sigma \approx \sqrt{E(\gamma + P)/a} \tag{10}$$

where:

P = Irreversible work absorbed in local plastic flow that was added to the surface energy term in the Griffith equation by Irwin and Orowan. The other terms are as previously mentioned.

Griffith assumed a crack (a) will propagate when the strain energy released (δ_E) per unit of crack extension exceeds the surface energy of the newly

formed crack. Strain energy released (δ_E) under plane-stress can be described for elliptical cracks as follows as determined by Ingliss:

$$\delta_E = \frac{\pi \sigma^2 a^2}{E} \tag{11}$$

where:

δ_E = the decrease in the strain energy in a plate per unit thickness as a result of crack length 2a under plane-stress. The crack is through the plate of unit thickness.

σ = applied stress required to propagate a crack (a)

a = 1/2 original crack length of unit thickness

The surface energy and irreversible work of plastic flow (α) of a crack of unit width W and 2a units long can be described as follows:

$$4a(\gamma + P)$$

where:

α = Total surface energy associated with the crack or potential energy of the crack/unit plate thickness and associated work of plastic flow.

γ = Surface energy per unit area or surface tension.

a = 1/2 the original crack or flaw length.

Griffith postulated that when small cracks are present in materials, spontaneous fracture would occur when the total energy of the system or structure was unchanged by small variations of the crack length as illustrated in the following energy balance equations:

$$\frac{\partial(\delta_E + \alpha)}{\partial a} = 0 \tag{12}$$

where:

δ_E = The decrease in strain energy in a system with a crack present.

α = The potential energy of the crack surface traction forces (surface energy) + dissipated energy of plastic flow. Orowan added plastic flow term.

$2a$ = The total length of the crack of unit width.

In other words, Griffith predicted sudden-fracture subcritically when

the energy release rate was equal to the rate of energy absorption due to the creation of the new surface. Therefore, the previously mentioned energy balance transforms to the following:

$$\frac{d(\delta_E + \alpha)}{da} = \frac{d}{da}\left|\frac{(\pi\sigma^2 a^2)}{E}\right| + 4a(\gamma + P) \tag{13}$$

$$\frac{2\pi\sigma^2 a}{E} = 4(\gamma + P) \tag{14}$$

$$\sigma = \sqrt{\frac{2(\gamma + P)E}{\pi a}} \tag{15}$$

For the essentially brittle materials which Griffith was considering, the energy associated with producing the new surface or increased crack length of unit width is in essence the surface tension. For metallic brittle materials, the energy associated with resistance to catastrophic cracking is surface tension and plastic deformation. The plastic work is difficult to obtain under test conditions. The determination of the surface energy of polycrystalline materials is relatively unexplored.

It appears that the projected ultimate practical applications would be the determination of the fracture toughness critical strength or critical flaw size of metallic materials from handbook surface tension data and determination of the energy associated with plastic work from a true stress-strain diagram test. However, surface energies of high strength alloys are not known and the depth of penetration of plastic strains would be needed.

Griffith assumed that a crack will propagate when the decreased stored strain energy term ($\frac{\pi\sigma^2 a^2}{E}$) exceeds the surface energy ($4a\gamma$) of the newly formed crack. The sign is negative of the decreased stored strain energy term, because it is the energy released in propagating the crack in the material. The total surface energy α of a crack of unit width and 2a units long should not be interpreted as γ (the surface energy per unit area) in Equation 12. The above theory predicts the strength reasonably well for bodies which behave in a quasi cleavage fashion such as glass and ceramics. However, the fracture of metals is accompanied by the dissipation of energy P by plastic flow adjacent to the fractured surfaces. This energy absorption (P) is often

at least 1000 times the energy corresponding to surface energy (γ) (Reference 9) for metals. Therefore, the stress parameter σ of the work term required to propagate a crack according to the Irwin-Orowan modification of the Griffith formula can be written as follows for metals:

$$\sigma = \sqrt{\frac{EPe}{a}} \tag{16}$$

Where:

Pe = an energy absorption factor at the tip of the crack where the strain energy accumulates to feed crack extension. $\gamma + P \rightarrow Pe$ where γ is ignored.

Plastic flow is always found at the edges of the fracture surfaces of metals. Therefore, the P term is the most important parameter for metals, but is next to impossible to measure and analytically evaluate in practice. As a consequence, the Fracture Mechanics Analytical Criteria was evolved by Irwin. In summary, the reason for the increasing speed of crack propagation according to both the Griffith's and Irwin-Orowan's equations is that once a crack has been initiated and the crack grows in length, the stress required for propagation continually decreases. In addition, when the environmental temperature is above the transition temperature of the metal, P is large and the stress σ required to make the crack grow will also be large. Below the transition temperature of the metal, the metal is brittle and P will be smaller. The stress necessary to cause crack growth, therefore will be reduced.

Orowan and Irwin independently proposed that the term in Griffith's equations be replaced by (γ + P), a term which excludes the surface energy but not the energy of plastic deformation, yielding (Reference 9):

$$a_c = \frac{2(\gamma + P)E}{\pi\sigma} \tag{17}$$

$$\sigma_c = \sqrt{\frac{2(\gamma + P)E}{\pi a}} \tag{18}$$

A new viewpoint was introduced by Irwin and Orowan who recognized that unstable crack extension would develop regardless of plastic flow, if the plastic strains tended to localize near the boundaries of the crack. As a consequence,

work expended in plastic deformation (P) replaced the surface tension (γ) as the factor controlling fracture toughness.

This was the beginning of "Fracture Mechanics." This trend of thought states that the strength of a high strength metal or alloy is a function of the load bearing capacity, σ, and the defect size, a. Metals and alloys are plastic materials, and there is a restriction of plastic action at the tip of a notch. The energy expended in propagating a crack equals approximately (γ + P). The plastic flow at the tip of the crack is of engineering significance because P is approximately 1000γ for metals (Reference 6).

The application of the Griffith model to metals and alloys is complicated by the following (Reference 6):

1. Metals and alloys undergo elastic-plastic response at the tip of a crack or notch.

2. Propagation energy is mainly a result of plastic flow and not the surface energy.

3. If a metal is fracture resistant or tough enough and (P + γ) high enough, the metal or alloy will go into gross yielding before a crack will propagate in an unstable manner causing catastrophic failure or fracture of the metal or alloy.

However, high strength metals and alloys with a strength-to-density > 700,000 inches have not been developed that will meet this structural criteria stipulated in 3 above.

(a) Orowan's Basic Mechanical Metallurgy Contribution Toward the Application of the Griffith Theory to the Fracture of Metals and Alloys

The commercially important high strength, structural aluminum alloys ($\sigma/\rho \geq$ 700,000 inches) are subject to brittle fracture. The low temperature brittleness can be attributed directly to this fact. When fracture occurs in polycrystalline high strength alloys by trans-crystalline cleavage, very little energy is expended in propagating the fractures, so they closely resemble those that occur in glass or other brittle isotropic elastic solids. Very little fundamental research has been done on the mechanics of quasi cleavage in polycrystalline metals. The most comprehensive study of crystalline cleavage has not been done on metal crystals, but on crystals of an ionic salt (LiF) by

Gilman (Reference 10). Much about the mechanics of fracture involving cleavage was done by Griffith on glass. There does not appear to be any evidence that anyone has observed true cleavage in a face-centered cubic metal. Because basal cleavage is a well-known phenomenon in zinc crystals, this does not mean that all hexagonal close-packed metals cleave on the basal plane as zinc crystals cleave. Magnesium does not cleave easily on the basal plane or any other plane, nor is there any information in the literature relative to cleavage of cadmium crystals (Reference 10). The one important group of metals in which cleavage is most frequently observed is the body-centered cubic metals. However, the alkali metals are body-centered cubic metals. They do not cleave. The cleavage plane in the body-centered lattice is usually the (100), although there are examples in which it has been indicated that cleavage along (110) is preferred (Reference 11). Cleavage fracture in crystals and polycrystals is often complicated by the fact that plastic deformation occurs around and ahead of the cracks as observed by Irwin and Orowan, independently.

However, Orowan attacked the problem from a mechanical metallurgy point of view as opposed to a pure continuum mechanics approach, which assumes a perfect lattice structure. Under the conditions of high crack-velocity and low temperature, plastic deformation tends to be supressed, and crystalline cleavage fractures are therefore, equivalent to "brittle fracture" materials such as cold glass. However, there is one important difference. In metals and alloys, cleavage tends to follow specific crystallographic planes of low indices. Whereas in glass, this condition has no meaning because the material is usually amorphous rather than being crystalline. There is some evidence that glass can deform plastically at room temperature (Reference 10). The amount of this deformation during the period of a brittle fracture is very small. One may assume that amorphous materials deform elastically to the instant of fracture (Reference 12). Therefore, the justification for the systematic exclusion of the assumption of any plastic contribution to the fracturing process is justified in the Griffith Theory for amorphous materials. Orowan initially assumed a truly elastic solid on an atomistic scale (one which fractures

without plastic flow of any kind) placed in a state of tensile stress by the induced stress (σ_N) and fracturing on the vertical plane A-A (fracture plane), as follows:

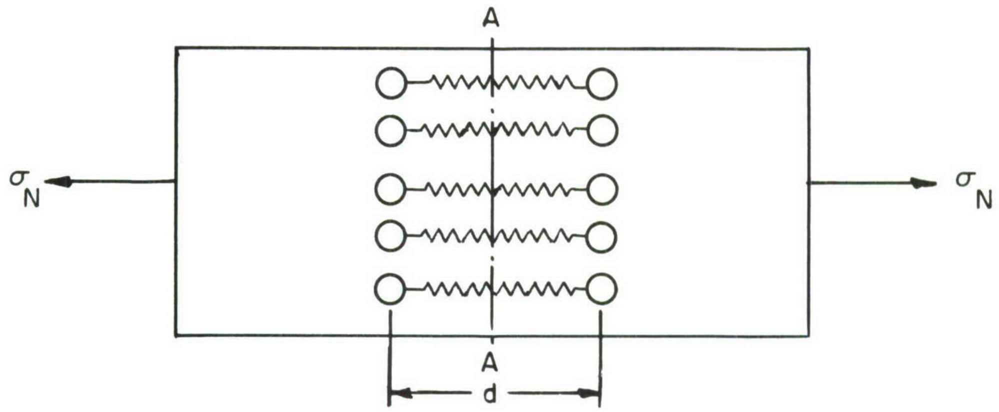


Figure 2. Orowan Model of Lattice Plane of a Crystal (Reference 10)

The two vertical rows of circles represent a pair of lattice planes with an interatomic distance spacing of d shown across the fracture plane A-A in Figure 2. Orowan assumed, if there were no flaws in the solid that fracture would occur by breaking bonds of distance d between atoms which face each other across the fracture plane A-A. In the model, the two vertical rows of circles are atoms, and the bond between the pair of atoms are indicated by short horizontal lines with a distance of d. The physical significance of this model is as follows:

(1) The bonds can be thought of as springs that permit atom pairs to be displaced toward or away from each other depending upon whether the applied external force that induces σ_N is compressive or tensile.

(2) For a small force and small strains, the atomic displacements will vary linearly with the applied stress (σ_N). Therefore, Hook's Law is applicable as follows:

$$\epsilon = \frac{X}{d} \tag{19}$$

$$\sigma_N = E\epsilon = \frac{EX}{d} \tag{20}$$

Where E = Young's modulus

ϵ = the strain

d = the interatomic distance in the absence of stress

σ_N = the stress normal to the fracture plane induced by a force

Orowan's model (Figure 3) for large displacements shows that the relationship between the displacement (X) and the applied stress σ_N is not linear (Reference 13).

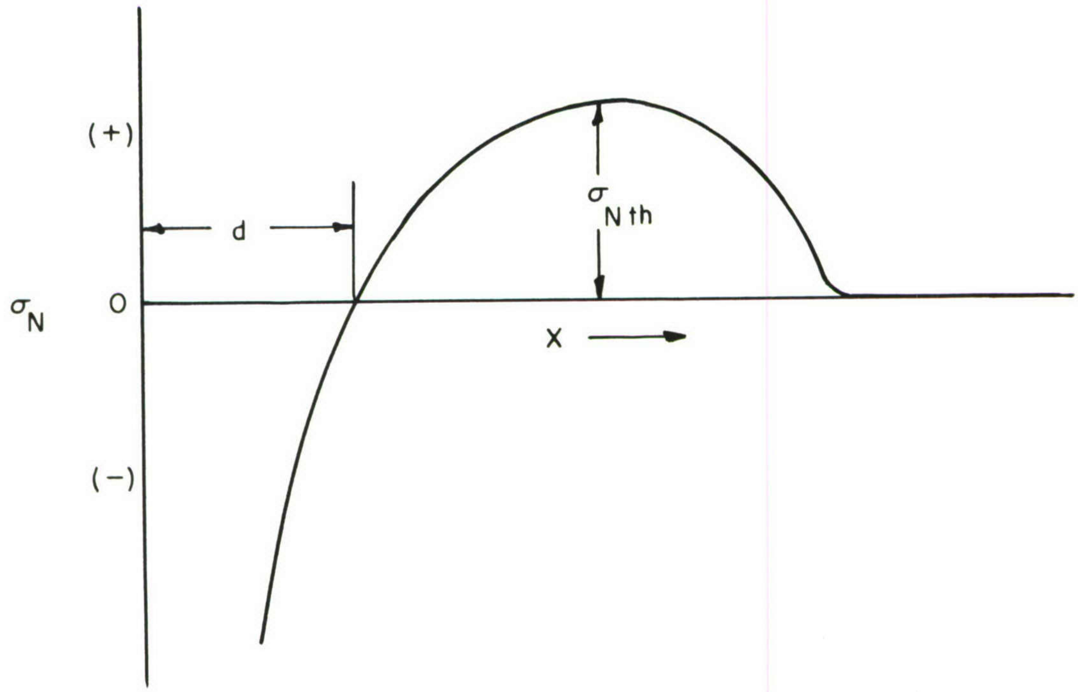


Figure 3. Cohesive Forces in a Solid as a Function of Interatomic Distance (Reference 13)

Therefore, Hooke's Law is not applicable. The mechanism of deformation and fracture from the above model can be explained as follows:

(1) When the atoms of Figure 2 are forced close together, very large repulsive forces are brought into play causing the magnitude of the compressive stress to rise more and more rapidly with increasing deformation as shown in Figure 3.

(2) When the atoms shown in the model in Figure 2 are displaced in the tensile direction, large positive displacements come into play causing a deviation from linearity. However, in this case the restoring force decreases at

a maximum point rather than grows in effectiveness as the displacement increases. This effect can also be deduced from Figure 3, which shows the curve that relates the stress induced and displacement on an atomic scale.

(3) From Figure 3, it can be readily seen that when the atomic bonds are lengthened, the restoring stress rises to a maximum and then falls.

(4) The value of the maximum stress (σ_{Nth}) is taken as the theoretical fracture stress.

(5) At the maximum stress (σ_{Nth}), a point of instability is reached where additional strain occurs under decreasing stress. With the normal method of loading in tension, this condition leads to fracture.

From Orowan's point of view (in the study of fracture), the interesting part of the curve in Figure 3 lies in the stress interval between 0 and σ_{Nth} (maximum restoring stress-intensity induced as a result of an external applied force). Inside the region of 0 to σ_{Nth}, the fracture curve (Figure 3) may be approximated as a simple sine function of the form (Reference 10):

$$\sigma_N = \sigma_{Nth} \sin \frac{2\pi X}{\lambda} \tag{21}$$

Where:

σ_N = the applied induced stress as shown in Figure 2

σ_{Nth} = the induced stress at the instant of fracture of the bond

X = the change in mean atomic distance or displacement

$\lambda/4$ = the value of X when the bond strength equal σ_{Nth}

The work per unit area (W) of the fracture surface which is expended in creating the fracture surface on the plane A-A in Figure 2 is:

$$W = \int_0^{\lambda/4} \sigma_{Nth} \sin \frac{2\pi X}{\lambda} \cdot dX = \frac{\sigma_{Nth}}{2\pi} \lambda \tag{22}$$

From Equation 21 which relates the stress to the displacement

(X), one may write for large values of X:

$$\frac{d\sigma_N}{dX} = \frac{d}{dX}\left(\sigma_{Nth}\sin\frac{2\pi X}{\lambda}\right) = \frac{2\pi}{\lambda}\,\sigma_{Nth}\cos\frac{2\pi X}{\lambda} \tag{23}$$

For small values of X, $\cos\frac{2\pi X}{\lambda} \approx 1$

Therefore:

$$\frac{d\sigma_N}{dX} = \frac{2\pi}{\lambda}\,\sigma_{Nth} \tag{24}$$

However, as previously stated for small displacements of X:

$$\sigma_N = E\,\frac{X}{d}$$

$$\frac{d\sigma_N}{dX} = \frac{E}{d} \tag{25}$$

Therefore, equating Equations 24 and 25 yields:

$$\frac{2\pi\sigma_{Nth}}{\lambda} = \frac{E}{d} \tag{26}$$

Solving for $\frac{\lambda}{2\pi}$ in Equation 26 and substituting it into Equation 22, yields the work per unit area as follows:

$$W = \sigma_{Nth}\,\frac{\lambda}{2\pi} = \frac{\sigma_{Nth}^2\,d}{E}\ \text{ergs/cm}^2 \tag{27}$$

When fracture occurs, this element of strain energy $\left(\frac{\sigma_{Nth}^2\,d}{E}\right)$ was assumed by Orowan to be transformed into the energy of the surfaces (2γ) that are created by the fracture.

Therefore:

$$2\gamma = \frac{\sigma_{Nth}^2\,d}{E} \tag{28}$$

$$\sigma_{Nth} = \sqrt{\frac{2\gamma E}{d}} \tag{29}$$

γ is the surface energy (surface tension), E is Young's modulus, and d is the mean interatomic distance across the fracture plane (shown in Figure 2 at zero stress).

The above equation (Equation 29) predicts extremely high theoretical fracture strength for isotropic elastic solids. Spretnak rationalizes that the failure to achieve the high σ_{Nth} is because perfect periodicity of the atomic structure does not exist because of the existence of structural imperfection such as line, point, and surface defects. Spretnak has indicated further that the onset of plastic flow is achieved (using Figure 3 as a reference) before the σ_{Nth} stress is reached, because of the imperfect atomic structure of the lattice (Reference 6). He stated that "travel up curve (Figure 3) is interrupted by onset of plastic flow because of presence of defects (point, line, and surface)." In many solids, d = 3Å, Young's modulus = 10^{11} dynes/cm^2, $\gamma = 10^3$ dynes/cm^2.

Therefore:

$$\sigma_{Nth} = \sqrt{\frac{2\gamma E}{d}} = \sqrt{\frac{2 \times 10^3 \times 10^{11}}{3 \times 10^{-8}}} = 10^{11} \text{ dynes/cm}^2$$

Tensile strengths of this order of magnitude are very rarely found. When obtained, they are found in freshly drawn glass fibers (Reference 14) and mica sheets loaded so as not to stress their edges (Reference 15). According to Orowan, mica ordinarily has a tensile strength of 30,000 to 40,000 PSI. Orowan assumed that this low strength was due to cracks located at the edges of the sheets. He further assumed that the flat surfaces of the sheets should be free of flaws since they are cleavage surfaces and should normally be without defects. In testing this hypothesis, Orowan made tensile test grips with a width smaller than the sheet width. In this manner, he loaded the mica sheet specimens so that their edges were not stressed. The result of this experiment was an observed tensile strength of more than 400,000 PSI (Reference 10).

Griffith's original work on glass fibers presented strong evidence for his theory. When freshly drawn soda-glass fibers were tested in bending, tensile strengths as high as 900,000 PSI were obtained, which is very close to the theoretical strength of glass ($\approx 10^6$ PSI). Griffith observed that the strength of glass is less than 1/100 of its theoretical strength (σ_{Nth}). This discrepancy

between the observed and the theoretical strength led Griffith to postulate that the low observed strengths were due to the presence of small cracks or flaws (a) in low-strength glass. Griffith assumed that the theoretical stress (σ_{Nth}) was obtained at the ends of a crack (a) even though the average stress (σ_N) was still far below the theoretical strength. This assumption was based upon the premise that the extremities of cracks have the ability to act as stress raisers. According to this concept, fracture occurs when the stress at the tips of the cracks exceeds the theoretical stress (σ_{Nth}). When this occurs, the crack is able to expand catastrophically (References 10, 14, and 16). Orowan (References 10 and 17) calculated the stress at the ends of a crack based upon the Griffith theory using a flat plate with an elliptical hole of the type shown in Figure 1. Ingliss (Reference 18) computed the stress and strain around an elliptical through-the-thickness crack of the type shown in Figure 1. Orowan et al expressed the theoretical fracture strength as follows:

$$\sigma_{Nth} = 2\sigma_N \left(\frac{a}{\rho} \right)^{1/2} \tag{30}$$

Where:

- $2a$ = the length of the major axis of the elliptical through-the-thickness crack
- σ_N = the average applied stress
- ρ = the radius of curvature at the ends of the elliptical crack

Orowan assumed that in an actual metal, the radius of curvature (ρ) at the tip of a crack is equal to the mean distance (d) between atoms across the fracture plane A-A shown in Figure 2.

Therefore:

$$\sigma_{Nth} = 2\sigma_N \left(\frac{a}{\rho} \right)^{1/2} = 2\sigma_N \left(\frac{a}{d} \right)^{1/2} \tag{31}$$

From Equation 29:

$$\sigma_{Nth} = \sqrt{\frac{2\gamma E}{d}}$$

Therefore:

$$\sigma_{Nth} = 2\sigma_N \left(\frac{a}{d}\right)^{1/2} = \left(\frac{2\gamma E}{d}\right)^{1/2} \tag{32}$$

Solving for the normal applied stress:

$$\sigma_N = \sqrt{\frac{\gamma E}{2a}} \tag{33}$$

Where:

σ_N = the average applied stress at which the crack will increase in length a

γ = the specific surface energy

2a = the through-the-thickness elliptical crack length

E = Young's modulus

The above relationship is Orowan's version of the Griffith criterion for brittle fracture. The numerical factor of Orowan's modification

$$\sigma_N = \sqrt{\frac{\gamma E}{2a}}$$

differ from Griffith criteria

$$\sigma_N = \sqrt{\frac{2\gamma E}{\pi a}}$$

by approximately 12 percent.

Again, it is observed from the two basic equations that as the crack length increases the stress (σ_N) to keep it moving decreases. This signifies that once the crack starts moving that it is able to accelerate to high velocities.

The above criteria of Griffith and Orowan applies specifically to a through-the-thickness flat elliptical crack lying in a flat plate. Calculations for cracks of other shapes in solids with different material geometries and more accurate atomic considerations have been made (Reference 10). The general functional relations shown in Equation 9 have been confirmed. Therefore, they can be considered of general significance with regard to "brittle fracture."

The above relationships and comparisons are applicable to elastic solids (such as cold glass). Therefore, the strain energy - $\frac{\pi \sigma^2 a^2}{E}$ that is released as a crack propagates is converted into the surface energy of the crack surfaces and the kinetic energy of the moving material at the sides of the crack. Orowan considered the additional energy term (P), (Equation 10) for the propagation of crystalline cleavage cracks. This term is the plastic deformation (P) occurring just ahead of the crack. The metal in this region is effectively in a state of very high uniaxial tensile stress with the axis of the force inducing the stress located normal to the plane of the crack. The energy of plastic deformation also comes at the expense of the elastic strain energy that drives the crack to create surface energy (γ) in the Griffith criteria. If the work to overcome plastic deformation is too large, the crack may not initiate the propagation stage. This implies that in those crystalline materials which cleave a minimum velocity must be achieved before the crack will propagate catastrophically. If the crack propagation velocity is slow, then the strain energy will be absorbed in the form of plastic flow (slip) predominately. This argument appears to set forth the premise that plastic deformation (P) is the most important factor which can be used to measure whether a metal is incapable of catastrophic fracturing or vice versa. The basic problem is how does plastic energy suddenly decrease at onset of running fracture.

Orowan's criteria are readily applicable to polycrystalline aggregate. As an approximation, one may consider that the average microcrack has a length equal to a grain diameter (D). The previously defined analogous Orowan criteria for the increase in crack length (a) for an induced applied stress

$$\sigma_N = \sqrt{\frac{\gamma E}{2a}}$$

may now be modified by assuming that the specific surface energy term (γ) can be replaced by a corresponding effective surface energy P_e. This latter quantity was used by Orowan to take into account within the fracture process the true surface energy and the energy of plastic deformation expended in

forcing the crack through a polycrystalline aggregate. The Reed-Hill version (Reference 10) of the Griffith criterion becomes

$$\sigma_N = \sqrt{\frac{(\gamma + P)E}{2a}} = \sqrt{\frac{P_e E}{2D}} \tag{34}$$

Where:

P_e = the effective specific surface energy

D = the average grain diameter

σ_N = the average applied stress

E = the Young's modulus

(b) Metallurgical Significance of the Orowan Criteria

The relation predicts that the strength of polycrystalline metals which fail by brittle fracture (cleavage) should vary as the reciprocal of the square root of the average grain diameter (Reference 10). Empirical relations of this nature have been reported for both zinc and iron (References 19 and 20).

The physical significance of

$$\sigma_N = \sqrt{\frac{P_e E}{2D}} = \sqrt{\frac{(\gamma + P)E}{2a}}$$

is that as the diameter of the grains becomes larger and larger, a critical diameter is reached above which the stress needed to increase the microcrack length becomes smaller than the stress to form or nucleate a crack inside a crystal. When this occurs, it is conceivable that the first crack that forms causes the failure of the specimen.

By the time the crack has grown to a size equal to the grain diameter, it will be large enough to pass through the boundary and continue to grow unless stopped by second phase particles. Therefore, above the critical diameter of the grains, the fracture stress is controlled by the stress required to nucleate cracks in crystals. Below the critical diameter, the fracture stress is controlled by the stress required to propagate cracks through a polycrystalline aggregate. Since cleavage cracks are not nucleated until the yield point of the

metal is reached, fracture should occur almost simultaneously with yielding at diameters larger than the critical diameter (Reference 10).

Gilman (Reference 21) has summarized the above conclusions in the form of a simple schematic diagram shown in Figure 4 (Summary of Experimental Results Showing the Effect of Grain Size D on the Yield and Fracture Stresses of Polycrystalline Metals, Reference 21).

Schematic stress-strain curves are shown in Figure 5 corresponding to the crack nucleation and propagation limited ranges shown in Figure 4. They are depicted to further rationalize the nucleation-propagation limitations of polycrystalline metals (Reference 10). Figure 5a of Figure 5 (Schematic Tensile Stress-Strain Curves) gives the shape of a curve that is nucleation-limited. Figure 5b illustrates fracture that is propagation-limited. There is small but definite plastic strain in the propagation-limited case. Figure 5c would be indicative of a high (γ + P) term with P $\gg$ γ. The high plastic deformation factor would make the metal incapable of failing by cleavage fracture, because slip would occur more easily. Therefore, the fracture mode would be shear (Figure 5c) as opposed to cleavage (Figures 5a and b).

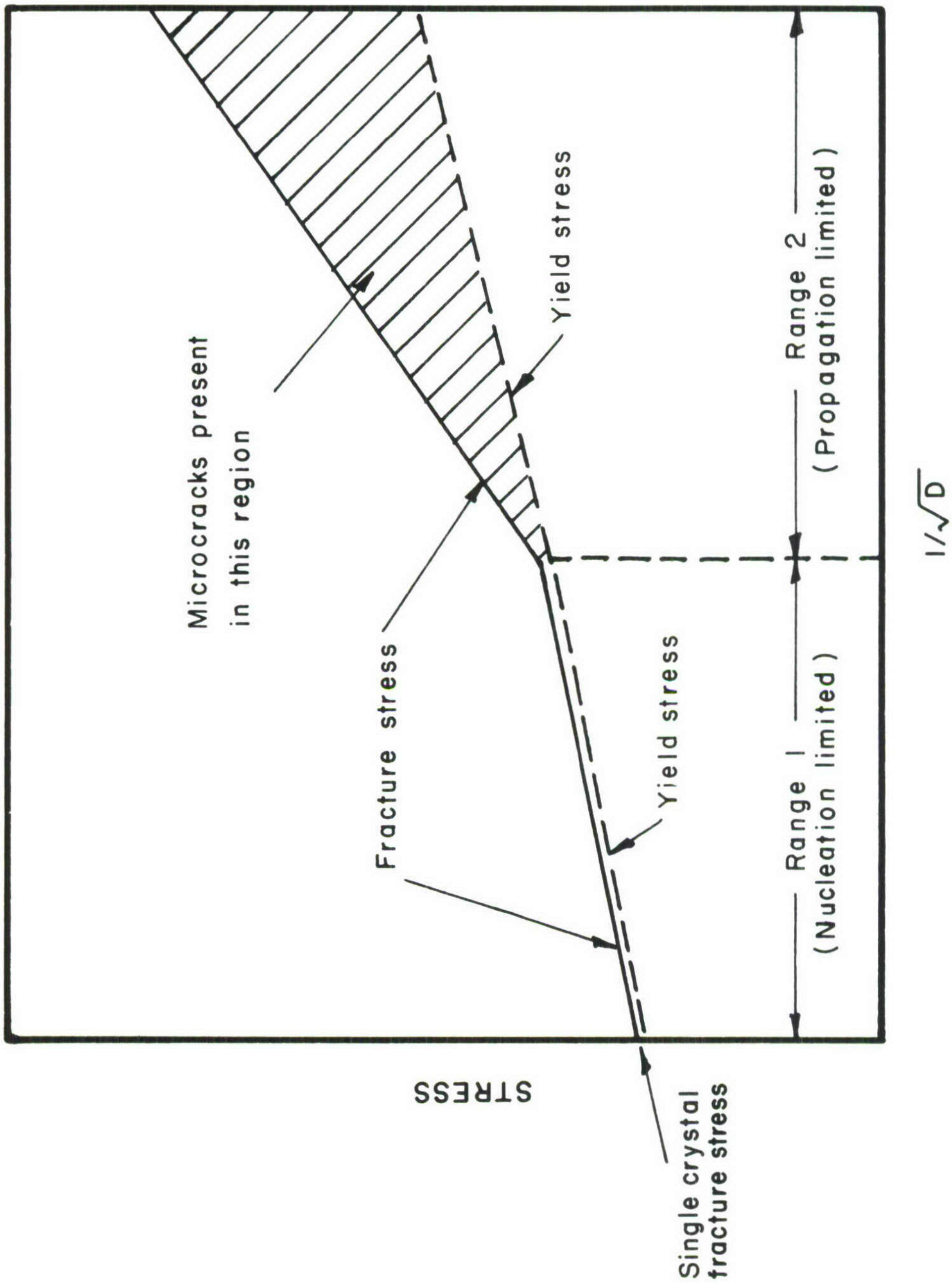


Figure 4. Summary of Experimental Results Showing the Effect of Grain Size D on the Yield and Fracture Stresses of Polycrystalline Metals. (After Gilman, J. J., Trans. AIME, 212, 1958, p. 783) (Reference 21)

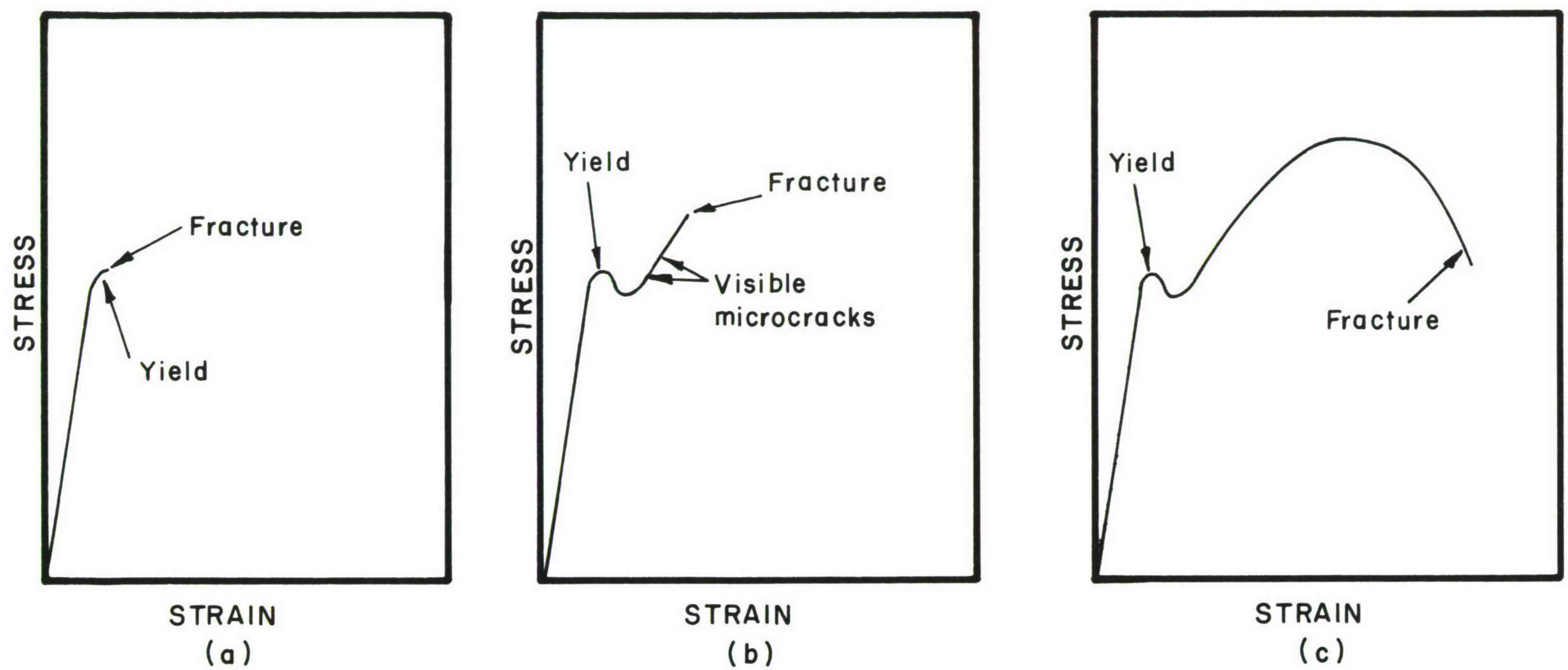


Figure 5. Tensile Stress-Strain Curves. (a) Brittle Fracture, Nucleation Limited; (b) Brittle Fracture, Propagation Limited; (c) Ductile Fracture (Reference 10)

(c) Irwin's Basic Mechanics Contribution Toward the Application of the Griffith Theory to the Fracture of Metals and Alloys

Irwin has written extensively on conditions giving rise to brittle fracture of metals based on a modification of the Griffith theory (Reference 9). The analysis is based upon the Griffith criterion modified for energy absorption by plastic flow as follows (Reference 9):

$$\sigma = \sqrt{\frac{P_e}{2a}} \tag{35}$$

σ = the critical fracture stress for a crack of size 2a

P_e = the surface energy and the energy of plastic deformation

E = the modulus of elasticity

a = 1/2 the crack length of an elliptical crack along the major axis

The analysis of the Irwin criterion of fracture mechanics involves two quantities defined by Wells as follows (Reference 9):

(1) Strain Energy Release Rate (G)

G is the quantity of stored elastic-strain energy released from a cracking specimen as a result of extension by a unit area of the advancing crack a.

(2) Fracture Toughness (G_c)

G_c is the component of work irreversibly absorbed in local plastic flow and cleavage surface tension to create a unit area of fracture. (Also referred to as Surface Energy Density, Fracture Extension Force.)

The Irwin concept of the "driving force per unit crack front" or the strain energy describing the physical significance of the "Strain Energy Release Rate (G)" and the "Fracture Toughness (G_c)" for high strength metals and alloys is a more generally applicable and a more readily useful concept than the Griffith theory. The Irwin concept does not require the assumption of the absence of plasticity and there is no complication involving fixed grips. It shows

that the "force tendency (G)" exists for any stress condition and that "G" can be determined from strain measurements near the crack tip. There are various formulas for "G". However, since applications of the basic "crack extension force tendency concept" to fracturing compared to the Griffith Theory are not widely known, a brief description of it is delineated subsequently in terms of elastic-plastic contributions. In conjunction, the critical value of the crack extension force G_c will be defined as the "fracture toughness" or the "force tendency" (crack extension force) to resist the onset of fast crack propagation in high strength alloys. The strain energy release rate is physically defined as follows (References 22, 23, and 24). Assume one has under consideration a solid object acted upon by external force, F_i, and the body contains a growing crack, a_i. If, during a time, δt, the work input from this external force is δW_E; the change is stored recoverable strain energy δU; and the difference causing the change in stored recoverable strain energy from δW_E to δU is δW_Q which consists primarily of a conversion of energy to heat within the solid object. Therefore, one can write a thermodynamic energy balance as follows:

$$\delta W_E = \delta U + \delta W_Q \tag{36}$$

δW_E can be expressed in terms of nonrecoverable and recoverable increments of motion of F_i since the work input δW_E is reduced to a stored recoverable strain energy δU by an amount equal to δW_Q.

Therefore:

$$\delta W_E = \sum_i F_i (\delta \ell_i^{P\ell} + \delta \ell_i^{e\ell}) \tag{37}$$

Therefore: $\delta \ell_i^{P\ell}$ and $\delta \ell_i^{e\ell}$ are the nonrecoverable and recoverable increments of motion of F_i, respectively.

The thermodynamic energy balance (Equation 36) has the following physical connotation in terms of the energy converted to heat within the body:

$$W_Q = \sum_i F_i \delta \ell_i^{P\ell} + \left[\sum_i F_i \delta \ell_i^{e\ell} - \delta U \right] \tag{38}$$

The elastic energy terms are $\sum_i F_i \delta \ell_i^{el}$ and δU. They are defined as the "Crack Extension Force Tendency" or the "Strain Energy Release Rate" as:

$$G = \frac{\sum_i F_i \delta \ell_i^{el} - \delta U}{\delta A} \tag{39}$$

This is the limiting case as δA, the element of new fracture area approaches zero. As deduced by Irwin, the only reason for this expression to differ from zero is the configurational change of the object due to extension of the crack a_i. The G value is assumed to depend only upon the applied forces, the configuration, and the elastic constants of the material. The omission of $\sum_i F_i \delta \ell^{P\ell}$ from the expression for the force tendency was a matter of choice by Irwin. However, his argument was that when one analyzes the longitudinal yielding of a tensile bar with no crack present, the G terms in the expression:

$$\frac{\delta W_Q}{\delta A} = \frac{\sum_i F_i \delta \ell_i^{P\ell} + \left[\sum_i F_i \delta \ell_i^{el} - \delta U\right]}{\delta A}$$

are of negligible importance compared to $\sum_i F_i \delta \ell_i^{P\ell} = F \delta \ell^{P\ell}$.

Therefore, the significant component of the "force tendency":

$$\delta W_Q = \sum_i F_i \delta \ell_i^{P\ell} + \left[\sum_i F_i \delta \ell_i^{el} - \delta U\right]$$

is

$$\delta W_Q = F \delta \ell^{P\ell} \quad \text{or} \quad F = \frac{\delta W_Q}{\delta \ell^{P\ell}} \tag{40}$$

for the case of the plastic "force tendency".

One may observe that on a unit area basis, this plastic "force tendency" is just the longitudinal stress.

Fundamentally, a rate problem is the primary concern. Therefore, the time rate of conversion to thermal energy is as follows:

$$\delta W_Q = \frac{F \delta \ell^{P\ell}}{\delta t} \tag{41}$$

The "force tendency" for a simple tension bar without a crack in it is depicted above and one would want to know the corresponding rate of yielding. This yielding rate has to be found experimentally. The familiar plot of $\frac{\delta \ell^{P\ell}}{\delta t}$ as a function of F shows the yielding rate increases from slow to fast over a relative small range of values of the "force tendency." However, work hardening effects were neglected in the plastic "force tendency" analysis. The value of F for this range per unit area is the yield strength and is properly regarded as an intrinsic material property.

Following an analogous procedure as delineated above for the longitudinal yielding of a tensile bar with no crack present, various amounts of fracture extension force

$$G = \frac{\sum_i F_i \, \delta \ell_i^{e\ell} - \delta U}{\delta A}$$

may be applied to a test specimen containing a crack and the response measured in terms of time rate of fracture extension. Generally, for high strength alloys the extension rate change is from slow to fast for relatively small changes of the crack extension force. The critical G values ($G = G_c$) have a significance relative to resistance to fracturing similar to that of the yield strength as a measure of resistance to yielding.

Irwin has shown that the crack extension force, G, is proportional to the square of the applied stress, σ, times the length of the crack, a, as one would anticipate from the Griffith analysis (References 7, 23, and 24). Therefore, for a central crack in a large plate subjected to tensile stress, σ, normal to crack direction, the crack extension force physical significance is as follows:

$$G = \frac{\sum_i F_i \, \delta \ell_i^{e\ell} - \delta U}{\delta A} = \frac{\pi \sigma^2 a}{E} \tag{42}$$

where 2a is the actual crack length and E is Young's modulus. This is the familiar Griffith expression previously derived in this review. Using the two-dimensional situation (in Figure 6a of Section III 1.a.(3) in which stresses are applied to a plate containing a crack along the X axis and such that τxy along the X axis is a zero (Reference 25), one can obtain a local stress interpretation of G. If the Cartesian position coordinates in Figure 6a of the

stress function are replaced by polar coordinates, r, θ, using one end of the crack as origin of coordinates (Figure 6a), as r approaches zero, the separational stress, σy, approaches

$$\sigma_y = \left(\frac{EG}{\pi}\right)^{1/2} \frac{\cos \theta/2}{2r^{1/2}} \left(1 + \sin\frac{\theta}{2} \sin\frac{3\theta}{2}\right) \tag{43}$$

The Westergaard-Irwin analysis treats the local stress interpretation of G in detail as summarized in Section III 1.a (3).

Irwin's hypothesis of the mechanism of the fracture process agrees with the Westergaard model shown in Figure 6a. Cracks extend by a process of development and joining up of new fracture origins near the crack tips. The comparative extension possibilities of cracks of different length and in different fields of overall nominal stress, σ, indicate that the development of new fracture origins proceeds under the influence of similar separational stress environments, σ_y, for two different cracks when the G for these cracks are the same. In summary, the physical significance of the Irwin criteria means that, in general, for cracks of various lengths and locations in any given solid material that the value of G measures the intensity of the crack-tip stress field so long as the influence of plastic deformation accompanying fracture extension is limited to the close neighborhood of the crack.

As a consequence, Irwin justifies the omission of $\sum_i F_i \delta \ell_i^{P\ell}$ from the expression of the total force tendency equation

$$\delta W_Q = \sum_i F_i \delta \ell_i^{P\ell} + \sum_i F_i \delta \ell_i^{e\ell} - \delta U$$

and consider the crack extension force tendency as

$$G = \frac{\sum_i F_i \delta \ell_i^{e\ell} - \delta U}{\delta A} = \frac{\pi \sigma^2 a}{E}$$

The condition of fracture is accordingly:

$$G = G_{Ic} \text{ (Plane Strain)}$$

$$G_{Ic} = \frac{K_{Ic}}{E} \left(1 - \nu^2\right) \tag{44}$$

Where K_{Ic} = stress-intensity factor at the tip of crack (a_{Ic}) under plane-strain conditions or the "fracture toughness" for high strength metals and alloys.

ν = Poisson's Ratio.

The premise is that a unit area of cleavage-type fracture of a given alloy at a given temperature absorbs the same amount of energy. The quantity G_{Ic} or K_{Ic} is considered a basic material characteristic (parameter) and consequently a possible design criterion to protect against brittle fracture.

Most of the experimentation has been directed to the determination of G_{Ic} of K_{Ic} for various high strength alloys. Test conditions for determining G_{Ic} must be such that a brittle fracture results. If slow crack propagation precedes a running crack some test method such as compliance, electrical potential or resistance, acoustical and/or photography must be employed to discern the area of crack initiation from slow crack propagation. To obtain values of G_{Ic} experimentally, it is necessary to have expressions for G in terms of crack (notch) dimensions, geometry of the test specimen, modulus of elasticity, Poisson's ratio, and stress field. Fracture toughness equations have been derived for infinite, semi-infinite and finite specimens for plane-strain measurements predominately. However, the center notch specimen can be used to measure the plane-strain, and plane-stress fracture toughness of high strength alloys if the optimum thickness and the maximum finite width is selected to assimilate the infinite plate's elastic-plastic stress distribution.

The practical and physical significance of the measure of the "fracture toughness" of a high strength alloy is essentially a measure of the ease and speed with which a crack propagates through the damaged material under selected stress and environmental conditions. Therefore, the mose direct measurement methods are those which measure the actual crack length as a function of stress and time. In the subsequent Section III 1.a.(3), a stress analysis criteria developed by Westergaard and supplemented by Irwin to determine the fracture resistance of metals and alloys is derived. The derived stress-intensity parameters are believed to be material properties that describe the toughness of high strength metals and alloys.

(3) Westergaard-Irwin Theoretical Stress Analysis Concepts and Techniques for the General Solution of the Plane-Strain and Plane-Stress Stress-Intensity Parameters of Cracks in the Griffith-Orowan-Irwin Model for Metals and Alloys

(a) Westergaard's Approach (Reference 26)

Westergaard's analysis technique for the general solution of problems of cracks in elastic sheets is based on a semi-inverse stress function method which involves the following mathematical model and general theory of elasticity. The problem considered is an infinite panel loaded in the x-y plane (Figure 6a). An airy stress function, ϕ, must satisfy the equilibrium and compatibility equations in solving for the normal and shear stresses acting on a differential element (dx dy) of material in the elastic stress field near the crack tip (Reference 25).

For equilibrium and a state of plane stress, $\sigma_z = 0$, the following is applicable:

$$\frac{\delta\sigma}{\delta x} + \frac{\delta\tau_{xy}}{\delta y} = 0 \tag{45}$$

$$\frac{\delta\sigma_y}{\delta y} + \frac{\delta\tau_{yx}}{\delta x} = 0$$

$$\tau_{yx} = \tau_{yx}$$

and the compatibility equation is:

$$\nabla^2(\sigma_x + \sigma_y) = 0$$

where

$$\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} \tag{46}$$

In addition, the normal and shear stresses are given in terms of ϕ as:

$$\sigma_x = \frac{\delta^2\phi}{\delta y^2} \; ; \; \sigma_y = \frac{\delta^2\phi}{\delta x^2} \; ; \; \tau_{xy} = -\frac{\delta^2\phi}{\delta x \delta y} \tag{47}$$

Combining Equations 45, 46, and 47 results in the biharmonic equation:

$$\nabla^4\phi = 0; \quad \text{where } \nabla^4 = \frac{\delta^4}{\delta x^4} + 2\frac{\delta^4}{\delta x^2 \delta y^2} + \frac{\delta^4}{\delta y^4} \tag{48}$$

By defining ϕ as:

$$\phi = \text{Re}\,\bar{\bar{Z}} + y\text{Im}\,\bar{Z} \tag{49}$$

where $\bar{\bar{Z}}$ is an analytic function of the complex variable

$$\zeta = x + iy$$

and

$$Z = \frac{d\bar{\bar{Z}}}{d\zeta}; \quad Z = \frac{d\bar{Z}}{d\zeta}; \quad Z' = \frac{dZ}{d\zeta} \tag{50}$$

results in $\nabla^4\phi = 0$. Equation 45 then reduces to:

$$\begin{aligned} \sigma_x &= \text{Re}\,Z - y\text{Im}\,Z' \\ \sigma_y &= \text{Re}\,Z + y\text{Im}\,Z' \\ \tau_{xy} &= -y\,\text{Re}\,Z' \end{aligned} \tag{51}$$

The stress function, $Z(\zeta)$, which solves the infinite panel problem of Figure 6a is

$$Z(\zeta) = \sigma\left[1 - \left(\frac{a}{\zeta}\right)^2\right]^{-1/2} \tag{52}$$

where σ is a uniformly applied tension stress. Along the line of expected crack extension (i. e., along the x axis, where $\theta = 0$, $y = 0$), the normal stress in the y-direction is:

$$\sigma_y = \text{Re}\,Z = \sigma\left[1 - \left(\frac{a}{\zeta}\right)^2\right]^{-1/2} \tag{53}$$

In addition (Reference 26), σ_y can be expressed as

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \tag{54}$$

Therefore, combining Equations 53 and 54 results in

$$K = \sigma \left[\frac{2\pi r \; x^2}{x^2 - a^2} \right]^{1/2} \tag{55}$$

where $x = a + r$. Since the stress-intensity factor describes the stress field near the crack tip, the following limiting process is taken:

$$K = \lim_{r \to 0} \sigma \left[\frac{2\pi (a + r)^2}{2a + r} \right]^{1/2} = \sigma \sqrt{\pi a} \tag{56}$$

Equation 56 is an infinite panel solution for the crack-tip stress-intensity factor K. However, it is of practical interest to determine the stress-intensity factor for a centrally cracked panel of finite width. The approach is to consider a collinear array of cracks of length 2a subjected to uniform tension, σ, and spaced at equidistant W as shown in Figure 6b. The appropriate Westergaard stress function that solves the given problem is (References 25 and 26):

$$Z(\zeta) = \frac{\sigma \sin \frac{\pi \zeta}{W}}{\left[\left(\sin \frac{\pi \zeta}{W} \right)^2 - \left(\sin \frac{\pi a}{W} \right)^2 \right]^{1/2}} \tag{57}$$

Therefore, along the x-axis:

$$\sigma_y = \mathrm{Re}\, Z = \frac{\sigma \sin \frac{\pi x}{X}}{\left[\left(\sin \frac{\pi x}{W} \right)^2 - \left(\sin \frac{\pi a}{W} \right)^2 \right]^{1/2}} \tag{58}$$

where $x = a + r$. The resulting stress-intensity factor is:

$$K = \lim_{r \to 0} \sigma \sin \frac{\pi (a + r)}{W} \left[\frac{2\pi r}{\left(\sin \frac{\pi (a + r)}{W} \right)^2 - \left(\sin \frac{\pi a}{W} \right)^2} \right]^{1/2} \tag{59}$$

or

$$K = \sigma \sqrt{\pi a} \left[\frac{W}{\pi a} \tan \frac{\pi a}{W} \right]^{1/2} \tag{60}$$

Therefore, Equation 60 is the stress-intensity factor for a finite centrally cracked panel loaded in uniform tension. In comparing Equations 56 and 60.

the term $\left[\frac{W}{\pi a} \tan \frac{\pi a}{W}\right]^{1/2}$ is a stress field correction factor for the finite geometry.

The previous stress analysis of the crack-tip stress field and stress-intensity factor for the centrally cracked fracture specimen configuration is based on mathematical elastic theory techniques. Therefore, certain corrections and limitations will be placed on the solution for the stress-intensity factor due to localized plastic deformation at the crack tip. Of primary interest is the size of the plastic enclave or zone at the tip of the crack. This is readily determined by applying a yield criterion and incorporating the stress field equations given previously (Reference 26). The yield criterion which gives results in good agreement with test results is the Henkyvon Mises distortional energy criterion (Reference 27). For the state of plane-stress or biaxial-stress condition, the yielding theory can be expressed as:

$$\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2 = \sigma_{YS}^2 \tag{61}$$

where σ_{YS} is the yield stress as determined from an axial test. Substitution of Equation 5 of Reference 26 into Equation 61 results in the following expression, which defines the extent of the plastic zone in terms of polar coordinates r and θ.

$$r = \frac{K^2}{2\pi\sigma_{YS}^2} \cos^2 \frac{\theta}{2} \left[1 + 3\sin^2 \frac{\theta}{2}\right] \tag{62}$$

Along the line of crack extension, i.e., $\theta = 0$ degrees, the width of the plastic zone,

$$W = r = \frac{K^2}{2\pi\sigma_{YS}^2} \tag{63}$$

The width of the plastic zone is considered as an added effective crack length. Therefore, Equation 60, when corrected for this additional crack length, becomes:

$$K_I = \sigma_I \left[W \tan\left(\frac{\pi a_I}{W} + \frac{K_I^2}{2W\sigma_{YS}^2} \right)\right]^{1/2} \tag{64}$$

Based on observations of experimental data, the ASTM Committee on Fracture Testing of High-Strength Materials (Reference 28) recommends that centrally cracked panels be so designed that $\sigma_n \leq 0.8\ \sigma_{YS}$, where σ_n is the net section failure stress.

(b) The Westergaard-Irwin Compliance Equation (Reference 29)

Westergaard (Reference 25) has provided a convenient two-dimensional stress analysis in rectangular coordinates of a very large flat plate with tension applied in the y direction and a system of cracks along the x axis each of length 2a and the center at x = 0, $\pm$L, $\pm$ 2L... as shown previously in Figure 6b.

The pertinent values are given in terms of an analytic function, of the complex variable z = x + iy, in Equation 57 and as follows:

$$Z = \frac{\sigma}{\sqrt{1 - \left(\frac{\sin\left(\frac{\pi a}{L}\right)}{\sin\left(\frac{\pi z}{L}\right)} \right)^2}} \qquad (65)$$

In Westergaard's notation $\bar{\bar{Z}}$, $\bar{Z}$, Z' are related by:

$$\bar{Z} = \frac{d\bar{\bar{Z}}}{dz}; \quad Z = \frac{d\bar{Z}}{dz} \text{ and } Z' = \frac{dZ}{dz}$$

and

Z' = Z(z) is an analytical function of the complex variable z = x + iy ϕ = Re $\bar{\bar{Z}}$ + y Im$\bar{Z}$ (Reference 26)

According to this method the stresses are assumed to be functions of x and y only and are expressed as:

$$\sigma_y = \mathrm{Re}\,Z + y\,\mathrm{Im}\,Z'$$

$$\sigma_x = \mathrm{Re}\,Z - y\,\mathrm{Im}\,Z'$$

$$\tau_{xy} = -y\,\mathrm{Re}\,Z'$$

and the displacement, v, in the y direction for a plane-strain situation

$$Ev = 2(1-\nu^2)\,\mathrm{Im}\bar{Z} - (1+\nu)\,y\,\mathrm{Re}\,Z \qquad (66)$$

where ν is Poisson's ratio and E is the modulus of elasticity.

Irwin has shown (Reference 30) that the above stress analysis is a good approximation to the stress situation in a centrally cracked sheet specimen (with vertical and horizontal axes of symmetry of the specimen as y and x axes). The specimen, with width W = L is considered to be one unit of the crack system (i.e., the plate with the system of cracks is regarded as being made up of a large number of centrally cracked specimens placed edge to edge). Thus the specimen edges are at x = $\pm$L/2 and the crack extends from x = -a to x = +a (Figure 6a).

Boundary conditions require the stresses σ_y and τ_{xy} to be zero along the borders of the crack and the stresses σ_x and τ_{xy} to be zero along the side boundaries. All of these conditions are fulfilled except the condition when σ_x = 0 along the side boundary. Irwin has remedied the situation by rewriting σ_x as

$$\sigma_x = \mathrm{Re}Z - y\,\mathrm{Im}Z' - \sigma_{ox}^{*}$$

where σ_{ox} is a constant stress adjusted so that

$$\int_0^{\infty} \sigma_x \, dy = 0 \quad \text{at } x = \pm L/2$$

The introduction of the constant σ_{ox} does not make σ_x zero everywhere along the side boundaries, but it does reduce σ_x to a small value. It is assumed that this deviation from an exact solution is smaller than the relatively small errors introduced in practical application by the departure from linear elasticity theory at finite strains.

Irwin modified the equation giving displacement in the y direction to make it applicable to a generalized plane-stress situation. The Westergaard-Irwin equation is

$$Ev = 2\,\mathrm{Im}\bar{Z} - (1+\nu)y\,\mathrm{Re}Z \qquad (67)$$

---

*Westergaard stated that an arbitrary constant stress could be applied in the x direction without disturbing the validity of the solution.

The Westergaard-Irwin equation shows an expression for compliance at the specimen edges. The conditions at the edge of the specimen are $x = \pm L/2$ so that $z = \pm L/2 + iy$ where y will be 1/2 of the specimen gage length.

The integration

$$\bar{Z} = \int Z \, dz$$

gives

$$\bar{Z} = -\frac{L\sigma}{\pi} \sin^{-1}\left[\frac{\cos \frac{\pi z}{L}}{\cos \frac{\pi a}{L}}\right] + C$$

where C is a complex constant of integration to be evaluated later using the known conditions at $a = 0$.

Making use of the identities:

$$\cos\left[\frac{\pi}{L}\left(\frac{L}{2} + iy\right)\right] = -i \sinh \frac{\pi y}{L}$$

$$\sin\left[\frac{\pi}{L}\left(\frac{L}{2} + iy\right)\right] = i \cosh \frac{\pi y}{L}$$

and

$$-\sin^{-1}(-im) = i \sinh^{-1}(m)$$

then

$$\bar{Z} = \frac{iL\sigma}{\pi} \sinh^{-1}\left[\frac{\sinh \frac{\pi y}{L}}{\cos \frac{\pi a}{L}}\right] + \mathrm{Re}C + i\mathrm{Im}C$$

so that

$$\mathrm{Im}\bar{Z} = \frac{iL\sigma}{\pi} \sinh^{-1}\left[\frac{\sinh \frac{\pi y}{L}}{\cos \frac{\pi a}{L}}\right] + \mathrm{Im}C$$

also $x = \underline{+}L/2$

$$Z = \frac{\sigma}{\sqrt{1 - \left(\dfrac{\sin \frac{\pi a}{L}}{\cosh \frac{\pi a}{L}} \right)^2}}$$

is real so that

$$\mathrm{Re}Z = Z \quad \text{at } x = \pm L/2$$

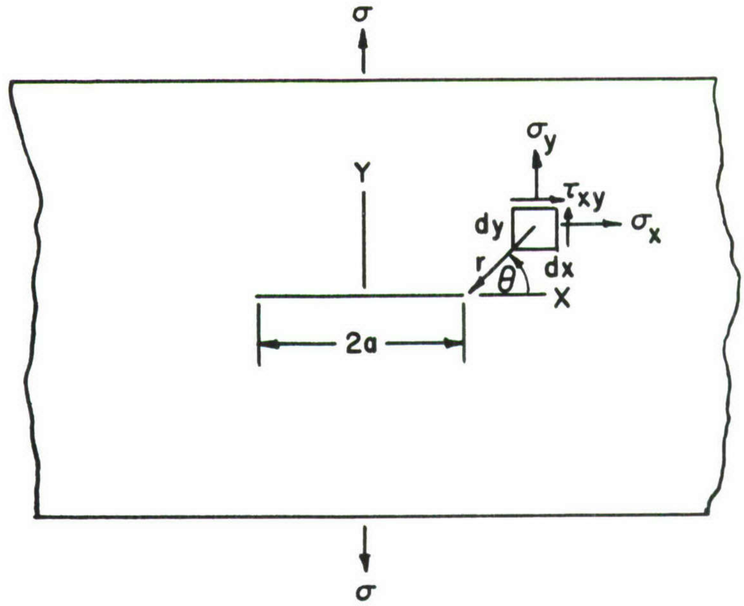
The expression for displacement v is found by substitution into Equation 67. Following Boyle (Reference 31) the compliance can be expressed in the dimensionless form $Ev/\sigma W$ by multiplying the expression by the modulus of elasticity and dividing by the specimen width w. Observing that $L = W$,

$$\frac{Ev}{\sigma W} = \frac{2}{\pi} \sinh^{-1} \left(\frac{\sinh \frac{\pi y}{W}}{\cos \frac{\pi a}{W}} \right) + \frac{2\mathrm{Im}C}{\sigma W} - \frac{y}{W} \frac{(1 + \nu)}{\sqrt{1 - \dfrac{\sin^2 \frac{\pi a}{W}}{\cosh^2 \frac{\pi y}{W}}}} \tag{68}$$

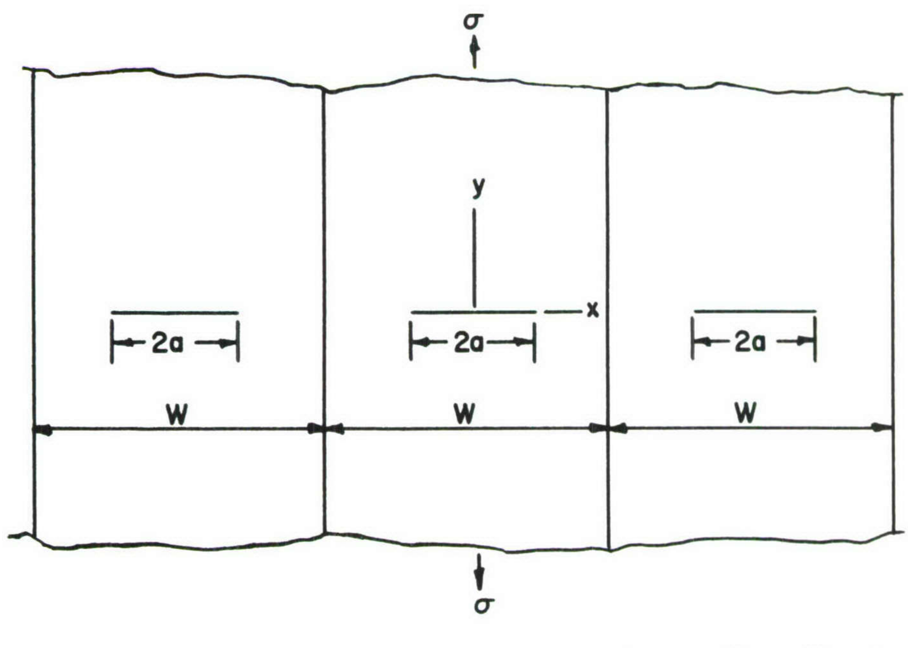
Im·C is evaluated by noting that at $a = 0$ the specimen corresponds to an unnotched specimen and $Ev/\sigma W$ is equal to y/W:

$$\frac{Ev}{\sigma W} = \frac{2}{\pi} \sinh^{-1} \left(\frac{\sinh \frac{\pi y}{W}}{\cos \frac{\pi a}{W}} \right) - \frac{y}{W} \frac{(1 + \nu)}{\sqrt{1 - \dfrac{\sin^2 \frac{\pi a}{W}}{\cosh^2 \frac{\pi y}{W}}}} + \frac{\nu y}{W}$$

Where $\frac{Ev}{\sigma W}$ is the mechanical compliance of a centrally cracked finite width specimen.



a. Infinite Panel Subjected to Uniform Tension



b. Collinear Array of Cracks Subjected to Uniform Tension

Figure 6. Centrally Cracked Panel Configuration with Infinite and Finite Width (Reference 2).

2. BOYLE'S ANALYTICAL AND EXPERIMENTAL RESULTS OF THE WESTERGAARD-IRWIN THEORETICAL COMPLIANCE OF THROUGH-THE-THICKNESS-CENTRALLY CRACK PLATE AND SHEET FOR THE DETERMINATION OF THE PLANE-STRAIN (K_{Ic}) AND PLANE-STRESS (K_c) FRACTURE TOUGHNESS STRESS PARAMETER OF HIGH STRENGTH ALLOYS (REFERENCE 2)

Boyle (Reference 31) has described a method of measuring the mechanical compliance (which is the extension-load ratio of a centrally notched or cracked sheet specimen) and has demonstrated how this measurement of compliance can be used to give a reliable estimate of notch depth plus cracks extending from the notch roots. The method is based on the fact that the compliance of a centrally cracked sheet is a function of the length of the crack (more accurately a function of the projection of the length of the crack on the transverse axis). As a crack extends, the compliance increases (stress now being taken as the gross stress calculated from the dimensions of the uncracked specimen) finally increasing without bounds as the crack reaches the edges of the specimen. As tension is applied to a centrally cracked sheet specimen, the extension of the specimen increases approximately linearly with applied gross stress until a stress level is reached at which crack extension occurs. As the crack extends, the slope of the load-extension curve decreases (load as ordinate) because of the increased compliance of the specimen. When the stress reaches an appropriate level, the specimen fails (the crack propagates rapidly across the remaining specimen width). The validity of the calculation of critical crack length depends on an assumption that the compliance value calculated from the slope of a line drawn from the origin of the load-extension curve to the point of discontinuity as shown in Figure 7 is the same as the compliance value of a specimen that contains an original crack the same length as the critical crack. Boyle proved this assumption valid (Figure 8).

In using the method described by Boyle (Reference 32), data from the compliance curve are used to calculate a numerical value for the dimensionless compliance factor:

$$\frac{Ev}{\sigma W}$$

E = the elastic modulus of the material.

v = the total displacement between the attachment points of the compliance gage (which are two inches apart) divided by 2.

σ = the gross stress.

W = the width of the specimen.

The numerical value for the compliance factor is selected on the ordinate of the calibration curve shown in Figure 8. This calibration curve was constructed by making load-deflection curves on a series of 3 by 12 inch slotted specimens of 7075-T6 aluminum alloy which were loaded only in the elastic range. The dimensional units used for the calibration curve permit the same curve to be used for other materials and any centrally cracked specimens in which the deformation is measured over a length equal to 2 W/3 where W is equal to the specimen width. The corresponding value for $\frac{\pi a}{W}$ can be obtained directly from the calibration curve, and this can be used in the equation for plane-stress tests in Figure 9 to determine G_c and/or K_c.

a. Plane-Stress Tests to Measure K_c

When sheet material is involved, fracturing at flaws is usually by mixed mode or by plane-stress mechanisms. In other words, there is considerable shear-lip development associated with the propagation of the fractures. In testing of precracked sheet specimens, the critical plane-stress condition occurs at the "onset of rapid crack propagation" which usually results after some "slow" crack development. For the stress analysis to be applicable, it is necessary to know the crack length at the start of rapid crack propagation at maximum load. The compliance gage discussed previously was developed for this purpose. A typical compliance-gage record for the mechanical type of gage is shown in Figure 7. The slope of the straight line that is the initial part of the compliance curve is dependent on the initial crack length as well as on the dimensions and elastic modulus of the specimen. Ideally, the slope of the line from the point of initiation of rapid fracturing to the origin would be dependent on the length of the crack at the point of instability as well as the dimensions and elastic modulus of the specimen. If the specimen is unloaded before fracturing, the trace of the unloading line will not go through the origin because of the slight amount of plastic deformation at the leading edge of the crack. However, if the slope of this line is "adjusted" to pass through the origin, it is assumed that this compensates for the effect of the plastic zone on the apparent crack length.

b. Plane-Strain Tests to Measure K_{Ic}

During development of the compliance gage, Boyle (Reference 32) noted that under certain conditions there was a sharp discontinuity or step in the compliance curve at the upper end of the initial straight-line portion. This step (Figure 7) represented rapid crack propagation with no increase in load until after the rapid crack propagation had stopped. Further examination indicated that it was representative of plane-strain fracturing (often referred to as crack opening mode or opening mode of crack extension) since it was a square-type fracture not associated with shear-lip formation. The load at the step in the curve is called the pop-in load, and the phenomenon is called meta-instability since it is not necessarily continuous. The equation for calculating the plane-strain parameters for the center-cracked specimens at the pop-in loads is shown in Figure 9.

According to the definition, the plane-strain fracture toughness is associated with unstable fracturing which produces a square fracture and a step or pop-in load (Figure 7).

A Boyle room temperature compliance gage which was fabricated at the AF Materials Laboratory and used by the author in a previous program (determination of the fracture toughness of several high strength steels) is depicted in Figures 10 and 11. The compliance gage was supplemented with a load-acoustical pickup system to increase sensitivity to plane-strain load pop-in and make it readily discernible for gross stress calculations for plane-strain fracture toughness material evaluation.

c. Basic Analytical and Experimental Problems Associated with the Determination of K_{Ic} and K_c

The basic analytical and experimental problems associated with the determination of K_{Ic} (plane-strain fracture toughness parameter) and K_c (plane-stress fracture toughness parameter) are:

1. The theory is based upon the concept of a finite crack propagating in an infinite plate under the influence of a uniformly applied tensile load and, in turn, setting-up within an elastically stressed body the stress-intensity of a force reaction from the tip of a crack outward to the boundaries, respectively.

Under these conditions the plane-strain stress-intensity factor (K_{Ic}) has been expounded by investigators to be a material constant independent of thickness of sheet or plate. Whereas, the plane-stress stress-intensity factor is dependent on the thickness of sheet or plate material.

2. There are basic experimental problems in the laboratory when one attempts to simulate the infinite plate K_{Ic} and K_c stress-intensity conditions from the tip of a crack to the boundaries in a finite laboratory test specimen.

3. The accuracy of laboratory measurements of K_{Ic} and K_c depends upon:

a. the adequacy of acuity of the starting crack.
b. the remoteness of the points of load application from the region of crack propagation.
c. the sufficiency of specimen size to provide the necessary elastic constraint around the plasticity developed at the tip of the crack prior and during noncritical crack propagation under plane-strain and plane-stress conditions.
d. the proper ratio relationship between crack length and finite specimen width in order that the nominal stress at the tips of the crack does not exceed prescribed maximum theoretical bounds derived for the finite specimen, so as to assimilate infinite specimen plastic-elastic conditions.
e. a sensitive method of detecting the onset of slow crack growth under plane-strain conditions of stress, the termination of slow crack growth under plane-stress and the simultaneously onset of catastrophic fracturing.

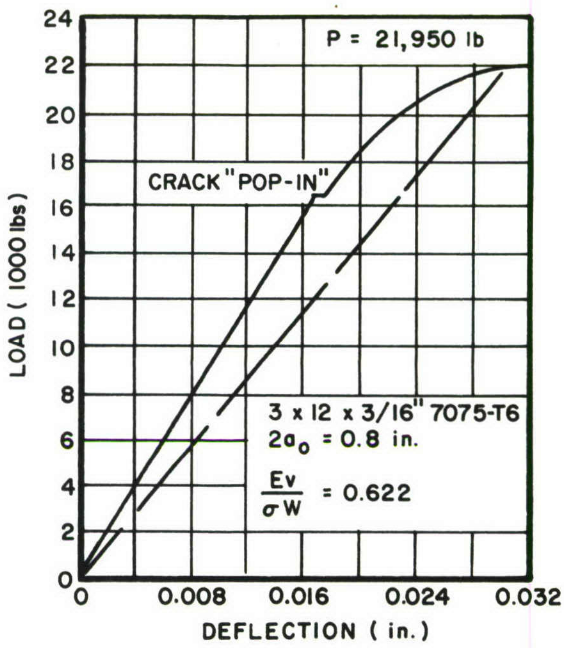
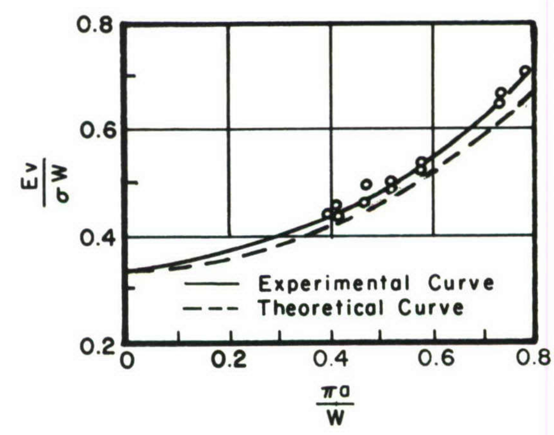
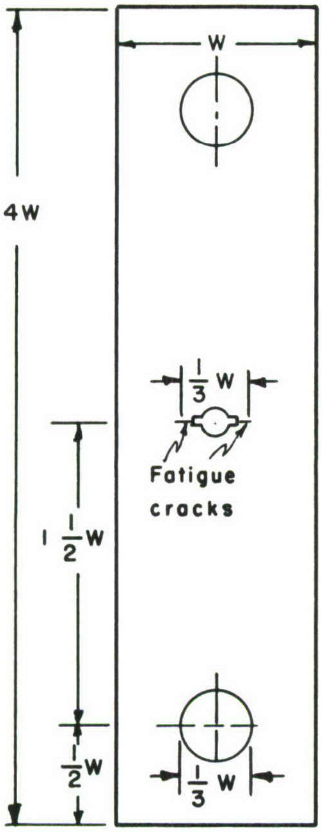


Figure 7. Typical Load-Deflection Curve for Centrally Cracked Sheet Specimens Showing Pop-In (Reference 32)



NOTE:
THEORETICAL CURVE IS FOR WESTERGAARD'S EQUATION

Figure 8. Calibration Curve Derived from Compliance Measurements on a Series of Specimens Having Various Slot Length (Reference 32)



FOR PLANE-STRAIN TESTS

Initial crack length $= \frac{1}{3} W = 2a_0$

$\frac{\text{Width}}{\text{Thickness}} \geq 8$

$EG_{Ic} = K_{Ic}^2(1 - \nu^2) = \sigma^2 W \tan \frac{\pi a}{W}$

ν = Poisson's ratio

σ = Gross stress at pop-in $= \frac{\text{Load in pounds}}{\text{W x Thickness}}$

E = Elastic modulus

a = One-half crack length at pop-in (usually $a_0 + \frac{EG_{Ic}}{6\pi\sigma_{YS}^2}$)

a_0 May be measured on the fracture

σ_{YS} = Yield strength of the material at 0.2 percent offset

$\frac{EG_{Ic}}{6\pi\sigma_{YS}^2}$ Is often neglected if it is very small compared to a_0

FOR PLANE-STRESS TESTS

Initial crack length $= \frac{1}{3} W = 2a_0$

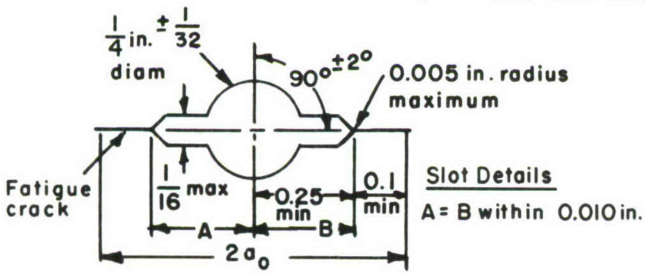
For $\frac{\text{Width}}{\text{Thickness}}$ greater than 16 and less than 45

$EG_c = K_c^2 = \sigma^2 W \tan \frac{\pi a}{W}$

σ = Gross stress at maximum load

$= \frac{\text{Maximum load in pounds}}{\text{W x Thickness}}$

a = One-half total crack length at maximum load $+ \frac{K_c^2}{2\pi\sigma_{YS}^2}$



Note: In one common specimen design for sheet and plate to 3/8-inch thickness, the width is three inches and the length is 12 inches.

Figure 9. Symmetrical Centrally Cracked Tensile Specimen for Sheet and Plate (Reference 33)

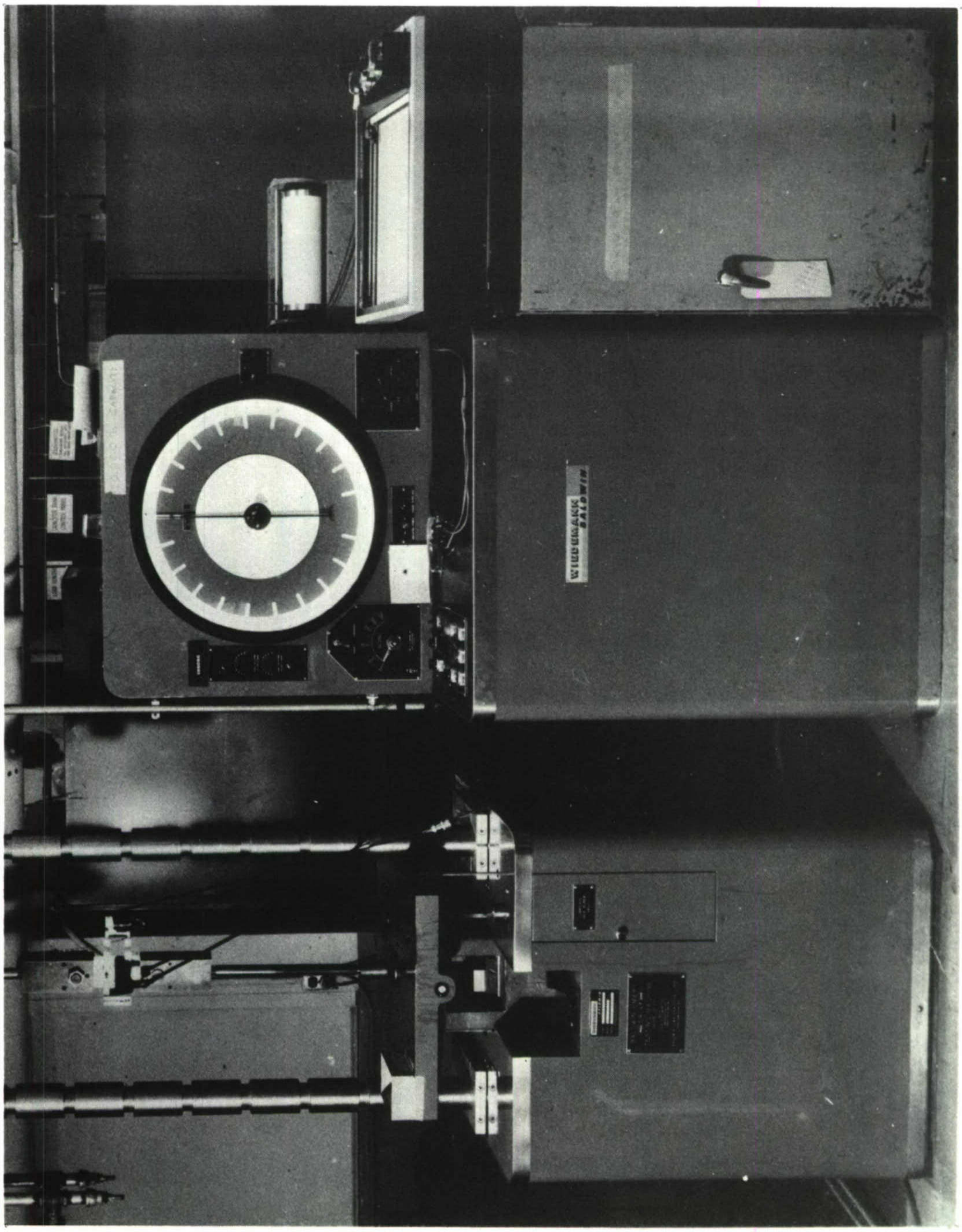


Figure 10. Boyle Room Temperature Compliance Gage Load-Extension System Combined with a Load-Acoustical Measurement System.

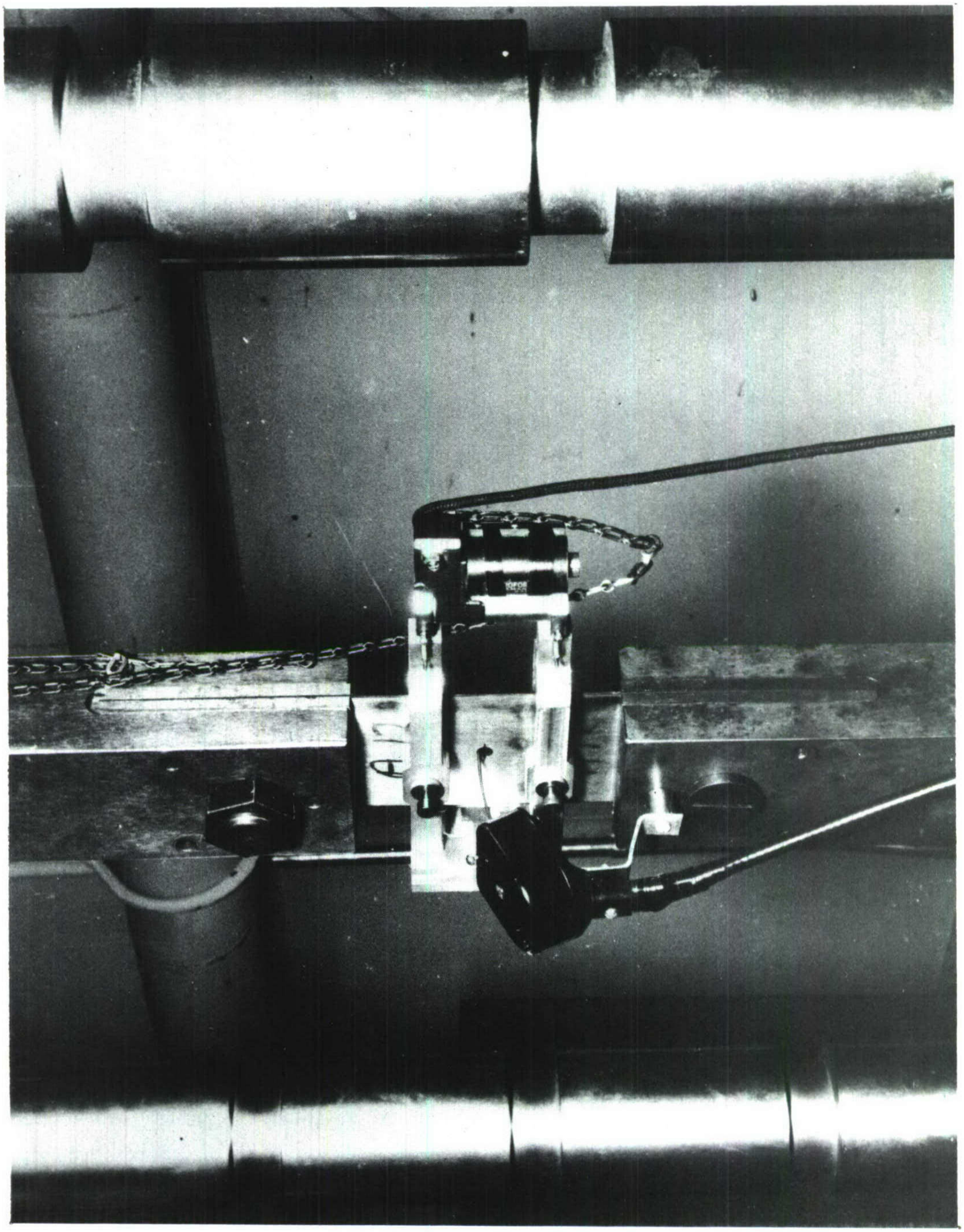


Figure 11. Detail View of Boyle Compliance Gage and Acoustical Pickup Crystal

SECTION IV

SUMMARY

A chronological and composite historical review of the development of concepts of linear elastic fracture mechanics from 1913 up to the present time has been documented for a reference as PART I of this technical report.

BIBLIOGRAPHY

1. Sidney O. Davis, Elevated Temperature Fatigue Properties of D6AC High Strength Steel, ASD-TDR-63-645, October 1963.

2. Sidney O. Davis, Development and Utilization of a Combination Cryogenic and Elevated Temperature Compliance Gage for Determination of Fracture Toughness of High Strength Metallic Materials, AFML-TR-66-335, July 1967.

3. "Fracture Testing of High-Strength Sheet Materials," A Report of a Special ASTM Committee, January 1960.

4. R. H. Christensen and P. H. Denke, Crack Strength and Crack Propagation Characteristics of High Strength Metals, ASD-TR-61-207, January 1962.

5. J. G. Kaufman, G. E. Nordmark, and B. W. Lifa, Fracture Toughness, Fatigue, and Corrosion Characteristics of 7075-T651, 7075-T7351, and 7079-T651 Aluminum Alloys, AFML-TR-65-170, May 1965.

6. J. W. Spretnak, Mechanical Metallurgy Lecture, 2 December 1965.

7. A. A. Griffith, "The Phenomenon of Rupture and Flow in Solids," Phil. Trans. Roy. Soc., London, A-221, 163-198, 1920.

8. W. Weibull, "A Statistical Theory of the Strength of Metals", Proceedings, Roy. Swed. Inst. Eng. Res., No.151, 1939.

9. J. W. Spretnak, A Summary of the Theory of Fracture in Metals, DMIC Report 157, August 1961.

10. Reed-Hill, Physical Metallurgy Principles, D. Van Nostrand Company, Inc., 1964, pp. 519-570.

11. C. S. Barrett and R. Bakish, Transactions AIME, 212, 1958, p. 122.

12. O. L. Anderson, Fracture, The Technology Press and John Wiley and Sons, Inc., New York, 1959, p. 331.

13. E. Orowan, Report on Progress in Physics, 12, 1948-1949, p. 185.

14. A. A. Griffith, Phil. Trans. Roy. Soc., A221 (1924), p. 163.

15. E. Orowan, Zeits, Fur Phys., 82 (1933), p. 235.

16. A. A. Griffith, First International Conference Appl. Mech., Delft (1924), 55A.

17. E. Orowan, Report on Progress in Physics, 12 (1948-49), p. 185.

BIBLIOGRAPHY (CONTD)

18. C. E. Ingliss, Inst. Naval Arch., 55 (1913), p. 219.

19. J. R. Low, Jr., Trans. ASM, 46A (1954), p. 163.

20. G. W. Greenwood and A. G. Quarrell, Journal Inst. of Metals, 82 (1954), p.551.

21. J. J. Gilman, Trans. AIME, 212, 1958, p. 783.

22. G. R. Irwin, "Fracture Dynamics," Fracture of Metals, (ASMS, Oct. 1947), A. S. M., Cleveland, 1948, Contains other papers.

23. G. R. Irwin, Onset of Fast Crack Propagation in High Strength Steel and Aluminum, WRL Report 4763, May 24, 1956.

24. G. R. Irwin, "Fracture," Pre-publication copy of Vol. VI article in Handbuch der Physik (Encyclopedia of Physics).

25. H. M. Westergaard, "Bearing Pressures and Cracks," J. Appl. Mech., Vol, 6, 2A, June 1939.

26. Air Force Materials Laboratory, Thick Section Fracture Toughness, ML-TDR-64-236, October 1964, pp. 15-17.

27. G. Murphy, Advanced Mechanics of Materials, McGraw-Hill Book Co., 1946, p. 61.

28. Fifth Report of a Special ASTM Committee, "Progress in Measuring Fracture Toughness and Using Fracture Mechanics," Materials Research and Standards, Vol. 4, No. 3, March 1964.

29. Morrison, Jenkins, and Kattus, Summary Technical Report on an Investigation of the Crack Propagation Resistance of High-Strength Alloys and Heat-Resistance Alloys, Southern Research Institute, November 1962, Bureau of Naval Weapons, Contract Now 61-0392-d.

30. G. R. Irwin, Fracture Testing of High-Strength Sheet Materials Under Conditions Appropriate for Stress Analysis, NRL Report 5486, July 27, 1960.

31. R. W. Boyle, An Instrument for Crack Growth Determinations from the Compliance of Notched Sheet Specimens, U.S. Naval Research Laboratory, Washington, D. C., Materials Research and Standards, Vol. 2, August 1962, p. 647.

32. R. W. Boyle, "A Method for Determining Crack Growth in Notched Sheet Specimens," Materials Research and Standards, Vol. 2, August 1962, p. 646.

BIBLIOGRAPHY (CONTD)

33. J. E. Srawley and W. F. Brown, Jr., "Fracture Toughness Testing," NASA Technical Memorandum, TM-X-52030, Technical Preprint Prepared for Symposium on Fracture Toughness Testing and Its Applications, Sponsored by the American Society for Testing and Materials at the Annual Meeting in Chicago, Illinois, June 23-24, 1964. (Presently "Fracture Toughness Testing and its Applications,") ASTM STP 31, April 1965, pp. 133-187.

34. F. A. McClintock and G. R. Irwin, "Plasticity Aspects of Fracture Mechanics," Advanced copy of paper for June 1964 ASTM Symposium on Crack Toughness Testing and Applications, Sponsored by the American Society for Testing and Materials at the Annual Meeting in Chicago, Illinois, June 23-24, 1964. (Presently "Fracture Toughness Testing and its Applications") ASTM STP 31, April 1965, pp. 84-109.

35. M. L. Williams, "Stress Distributions at the Base of a Stationary Crack," J. Appl. Mech. 24, p. 104 (1957).

36. G. R. Irwin, "Relation of Stresses Near a Crack to the Crack Extension Force," Paper No. 101 (II), IXth Int. Congress Appl. Mech., Brussels (1956).

37. G. R. Irwin, "Fracture," Springer "Encyl. of Physics," VI, p. 551 (1958).

38. J. A. H. Hult and F. A. McClintock, "Elastic-Plastic Stress and Strain Distribution Around Sharp Notches Under Repeated Shear," IXth Int. Congress Appl. Mech. 8, p. 51, Brussels (1956).

39. M. F. Koskinen, "Elastic-Plastic Deformation of a Single Grooved Flat Plate Under Longitudinal Shear," Trans. ASME, 85, D, p. 585 (1963).

40. Paul C. Paris and George C. M. Sih, "Stress Analysis of Cracks," American Society for Testing and Materials, Authorized reprint from the copyrighted Symposium on Fracture Toughness Testing and its Applications, Special Technical Publication No. 381, 1965.

41. J. E. Neimark, "The Initiation of Ductile Fracture in Tension," Sc. D. Thesis. Department of Mechanical Engineering, Massachusetts Institute of Technology, 1959.

42. R. Hill, "On Discontinuous Stress States with Special Reference to Necking in Thin Sheets," Journal of Mechanics and Physics of Solids, 1, p. 19 (1952).

43. A. P. Green, "The Plastic Yielding of Metal Junctions Due to Combined Shear and Pressure," Journal of Applied Mechanics and Physics of Solids, 2, 197 (1955).

BIBLIOGRAPHY (CONTD)

44. J. C. Walter, "Cracking from Notches Under Transverse Shear," M. Sc., Thesis. Department of Mechanical Engineering, Massachusetts Institute of Technology (1959).

45. S. S. Rhee and F. A. McClintock, "On the Effects of Strain Hardening on Strain Concentration," Proceedings of the 4th National Congress of Applied Mechanics, p. 1007, Berkeley (1962).

46. A. A. Wells, "Application of Fracture Mechanics at and Beyond General Yielding," British Welding Research Association Report M13/63, British Welding Journal, Nov. 1963, pp 563-570.

47. J. B. Walsh and A. C. Mackenzie, "Elastic-Plastic Torsion of a Circumferentially Notched Bar," Journal Mech. and Physics of Solids, 7, p. 247 (1959).

48. F. A. McClintock, "On the Plasticity of the Growth of Fatigue Cracks," Fracture of Solids, (D. C. Drucker and J. J. Gilman, Ed.) J. Wiley and Sons, p. 65 (1963).

49. F. A. McClintock, "Ductile Fracture Instability in Shear," Journal Appl. Mech. 25, p. 582 (1958).

50. F. A. McClintock, "Discussion of Fracture Testing of High Strength Sheet Materials," Materials Res. and Standards 1, p. 277 (1961).

51. G. R. Irwin, M. F. Koskinen, Discussion and author's closure to reference (5), Trans. ASME, 85, D, p. 590 (1963).

52. G. R. Irwin, "Relation of Crack Toughness Measurements to Practical Applications," Welding Journal Res. Suppl. (Nov. 1962).

53. G. R. Irwin, "Crack Toughness Testing of Strain Rate Sensitive Materials," ASME Mtg., (Phila. Nov. 1963) Trans. ASME (forthcoming), Naval Research Laboratory, Washington, D. C.

54. F. A. McClintock, "On Notch Sensitivity," Welding Journal Res. Suppl. 26, p. 202-s (1961).

55. D. S. Dugdale, "Yielding of Steel Sheets Containing Slits," Journal of Mech. and Physics of Solids, 8, p. 100 (1960).

56. F. A. Field, "Yielding in a Cracked Plate in Longitudinal Shear," Trans. ASME, 30, E, p. 622 (1963).

57. B. A. Bilby, A. H. Cottrell, and K. H. Swinden, "The Spread of Plastic Yield from a Notch," Proc. Roy. Soc. A, Vol. 272, p. 304 (1963).

BIBLIOGRAPHY (CONTD)

58. J. J. Gilman, "Cleavage and Ductility in Crystals," Proc. Int. Conf. on Atomic Mechanisms of Fracture, Techn. Press (MIT) and J. Wiley and Sons, p. 193 (1959).

59. "Fracture Testing of High Strength Sheet Materials," 1st Report of Special ASTM Comm., ASTM Bulletin, Jan. 1960.

60. J. A. H. Hult, "Fatigue Crack Propagation in Torsion," Journal of Mech. and Phys. of Solids, 6, p. 47 (1957).

61. F. A. McClintock, "Numerical Integration of Shear Cracking Equations, II" Res. Mem. No. 8, Dept. Mech. Eng., Mass. Inst. of Techn. (Nov. 15, 1958).

62. F. A. McClintock, "A Criterion for Ductile Fracture by Growth of Holes," submitted for publication (1965), Mechanical Engineering Department, Massachusetts Institute of Technology, Cambridge, Mass.

63. J. M. Krafft, "Correlation of Plane-Strain Crack Toughness with Strain Hardening Characteristics of a Low-, a Medium-, and a High-Strength Steel," Appl. Materials Res. (April 1964).

64. Fifth Report of Special ASTM Comm. on Fracture Testing of High Strength Materials, Materials Res. and Standards (March 1964) p. 107.

65. J. R. Lawson, "Crack Surface Displacements for an Elastic Material Under Longitudinal Shear," Res. Me. No. 46, Fatigue and Plasticity Lab., Dept. of Mech. Eng., Mass. Inst. of Techn. (1963).

66. R. W. Boyle, "A Method for Determining Crack Growth in Notched Sheet Specimens," Materials Res. and Standards 2, p. 646 (1962).

67. G. R. Irwin, "Plastic Zone Near a Crack and Fracture Toughness," 7th Sagamore Ordn. Materials Res. Conf. (Aug. 1960).

68. B. M. Wundt, "Unified Interpretation of Room Temperature Strength of Notched Specimens as Influenced by Their Size," ASME paper 59-MET 59, (1959), ASME, United Engineering Center, N. Y., N. Y.

69. J. M. Krafft and G. R. Irwin, "Crack Velocity Considerations," ASTM Symposium on Fracture Testing and its Applications, Chicago, 23 June 1964.

70. G. R. Irwin, "Fracture," Encycl. of Physics (Springer) Vol. VI, pp 551-590 (1958).

71. E. H. Yoffe, "The Moving Griffith Crack," Phil. Mag. 42, p. 739 (1951).

BIBLIOGRAPHY (CONTD)

72. J. A. Kies, A. M. Sullivan, and G. R. Irwin, "Interpretation of Fracture Markings," Journal Appl. Phys. 21, p. 716 (1950).

73. R. W. Boyle, A. M. Sullivan, and J. M. Krafft, "Determination of Plane Strain Fracture Toughness with Sharply Notched Sheets," Welding Journal Res. Suppl., Vol. 27, No. 9 (Sept. 1962), p. 428.

74. G. R. Irwin, "Crack Toughness Testing of Strain Rate Sensitive Materials," ASME Paper 63-WA-217, Nov. 1963., ASME, United Engineering Center, N. Y., N. Y.

75. J. M. Krafft, A. M. Sullivan, and R. W. Boyle, "Effect of Dimensions on Fast Fracture Instability of Notched Sheets," Crack Propagation Symposium, Cranfield, 1961.

76. G. R. Irwin, Discussion of paper by M. F. Koskinen, Trans. ASME, 85, Series D, p. 591 (1963).

77. J. M. Krafft and A. M. Sullivan, "Effects of Speed and Temperature on Crack Toughness and Yield Strength in Mild Steel," ASM Trans. Quarterly, Vol. 56, No. 1, March 1963, p. 160.

78. Progress in Measuring Fracture Toughness and Using Fracture Mechanics, Matls. Research and Standards, Vol. V, No. 3, March 1964, p. 107.

79. J. M. Krafft, "Correlation of Plane Strain Crack Toughness with Strain Hardening Characteristics of a Low, a Medium and a High Strength Steel," Applied Materials Research, Vol. 3, p. 88, April 1963.

80. F. A. McClintock and J. A. H. Hult, "Elastic-Plastic Stress and Strain Distributions Around Sharp Notches in Repeated Shear," IXth Int. Congress on Appl. Mech., Brussels (1956).

81. C. D. Beachem, "An Electron Fractographic Study on the Influence of Plastic Strain Conditions Upon Ductile Rupture Processes in Metals," Trans. Am. Soc. Metals, Vol. 56, Sept. 1963, p. 318.

82. C. Zener and J. H. Hollomon, "Effect of Strain Rate Upon Plastic Flow of Steel," Journal Applied Physics, Vol. 15, Jan. 1944, p. 22.

83. T. A. Read, H. Marcus, and J. McCaughey, "Plastic Flow and Rupture of Steel at High Hardness Levels," Fracturing of Metals, ASM, Cleveland, 1948.

84. G. R. Irwin, "Crack-Toughness Testing of Strain-Rate Sensitive Materials," U.S. Naval Research Laboratory, Washington, D. C. (Prepublished Paper).

BIBLIOGRAPHY (CONTD)

85. J. M. Krafft, "Fracture Toughness of Mild Steel," Note for ASTM-F THSM Meeting, Washington, D. C., Dec. 17, 1963.

86. W. J. Nordell and W. J. Hall, Two-Stage Fracturing in Welded Mild Steel Plates, Report, Oct. 1963, University of Illinois, Urbana, Illinois.

87. A. M. Freudenthal, The Inelastic Behavior of Engineering Materials and Structures, John Wiley and Sons, Inc., New York (1950) p. 74.

88. A. E. H. Love, Mathematical Theory of Elasticity, Cambridge Press, 4th Edition, 1927.

89. W. T. Lankford, J. R. Low, and M. Gensamer, "The Plastic Flow of Aluminum Alloy Steel Under Combined Loads," Metals Technology, AIME T. P. 2237, August 1947.

90. C. Zener and J. H. Holloman, "Effect of Strain Rate Upon Plastic Flow of Steel," Journal of Applied Physics, 15, 22 (1944).

91. J. H. Holloman, The Problem of Fracture, Welding Society Monograph, New York, 1947.

92. D. J. McAdams and R. W. Mebs, The Technical Cohesive Strength and Other Mechanical Properties at Low Temperatures, Proc. ASTM, 43, 661-703 (1943).

93. E. Orowan, " Low Temperature Plasticity," Zeitschrift fur Physik, 89, 605-659 (1934).

94. R. C. Meacham, Theories of Plastic Deformation of Single Crystals, Report A11-53 N7 Onr-358 , Naval Research Laboratory, Washington, D. C.

95. H. J. Holloman and C. Zener, "Conditions of Fracture of Steel," Trans. AIME 158, 283 (1944).

96. George E. Dieter, Jr., Mechanical Metallurgy, McGraw-Hill Book Company, Inc., New York 1961, pp. 24, 27.

97. Maxwell Gensamer, " Strength of Metals Under Combined Stresses," American Society for Metals, Cleveland (1941), pp.10, 79.

98. K. Heindlhofer, "Plasticity of Iron at Low Temperatures," Trans. Amer. Inst. Min. Met. Engrs., 116, 232-238 (1935).

99. Clarence Zener, "The Micro-Mechanism of Fracture, Fracturing of Metals," Amer. Soc. for Metals, Cleveland (1948), p. 3.

BIBLIOGRAPHY (CONTD)

100. A. H. Cottrell, "Theoretical Aspects of Fracture," Fracture, edited by B. L. Averbach, D. K. Felbeck, G. T. Hahn, and D. A. Thomas, The Technological Press of MIT and John Wiley and Sons, Inc. (1959) p. 20.

101. C. J. Beevers and R. W. K. Honeycombe, "Ductile Fracture of Single Crystals," Reference 100, p. 474.

102. F. P. Rybalko, "Plastic Strain and Fracture of Single Crystals and Polycrystals of Metals Under Static and Cyclic Torsion," Fiziki Metallov i Metallovedenie, Vol. 1, No. 2, 1955, pp. 231-238.

103. N. J. Petch, "The Fracture of Metals," Progress in Metal Physics, Vol. 5, Pergamon Press Ltd., London, pl (after M. Polyani, E. Orowan, F. Seitz, and T. A. Read).

104. B. Epstein's general treatment in J. Applied Physics, 19, 140 (1948).

105. C. F. Tipper, "The Brittle Fracture of Metals at Atmospheric and Sub-Zero Temperatures," Metallurgical Reviews, 2 (7), (1957).

106. M. Moser, "A New Method of Interpreting Notch-Bar Impact Test Results," Trans. Am. Soc. Metals, 1, 297-320 (1925).

107. E. T. Wessel, "Some Exploratory Observations of the Tensile Properties of Metals at Very Low Temperatures," Trans. Am. Soc. Metals, 49, 149-169 (1957).

108. F. Forscher, "A Theory of the Yield Point and the Transition Temperature of Mild Steel," Presented at the Annual Meeting of ASME, Chicago, November 13-18, 1955 , ASME, United Engineering Center, N. Y., N. Y.

109. J. H. Holloman, "The Notch-Bar Impact Test," AIME T. P 1667 (1944).

110. A. J. Kontorova, "On the Problem of Critical Temperature of Embrittlement," J. Techn. Physics USSR (in Russia), 9 (12) (1939).

111. A. H. Cottrell, "Theory of Brittle Fracture in Steel and Similar Metals," Trans. Amer. Inst. Min. Met. Engrs., 212, p.192 (1958).

112. N. J. Petch, "The Ductile-Cleavage Transition in Alpha-Iron," Reference 117, pp. 161-180.

113. N. Lovat and H. L. Wain, "Brittle Fracture and the Yield-Point Phenomenon," Reference 117, pp. 161-180.

114. I. R. Kramer and R. Maddin, "Delay Time for the Initiation of Slip in Metal Single Crystals," J. of Metals, pp. 2-8 (February, 1952).

BIBLIOGRAPHY (CONTD)

115. Tien-Shih Lin, I. R. Kramer, and M. A. Steinberg, "The Delay-Time Phenomenon in Metal Single Crystals," Acta Metallurgical, 4 (4), pp. 364-370 (July 1956).

116. John C. Fisher, "Application of Cottrell's Theory of Yielding to the Delayed Yield in Steel," Trans. Am. Soc. Metals, 47, pp. 451-462 (1955).

117. Fracture (Swampscott Conference), edited by B. L. Averbach, D. K. Felbeck, G. T. Hahn, and D. A. Thomas. Published jointly by the Technology Press of the Massachusetts Institute of Technology and John Wiley and Sons, Inc., New York (1959).

118. G. R. Irwin, "Fracture Dynamics, Fracturing of Metals," American Society for Metals, Cleveland (1948), p. 147.

119. G. M. Boyd, "The Conditions for Unstable Rupturing of a Wide Plate," Presented at the Spring Meeting of the Institute of Naval Architects, London, March 28, 1957.

120. N. F. Mott., Engineering, 165, (1948), p. 16.

121. O. L. Anderson, Fracture, The Technology Press and John Wiley and Sons, Inc., New York, 1959, p. 331.

122. D. K. Roberts and A. A. Wells, Engineering, 178 (1955), p. 820.

123. Robert E. Reed-Hill, Physical Metallurgy Principles, D. Van Nostrand Company, Inc., New York (1964), p. 531.

124. S. S. Manson, Fatigue: A Complex Approximations, Experimental Mechanics, Vol. 5, No. 7, July 1965 ("The William M. Murray Lecture, 1964").

125. R. H. Christensen, The Fatigue of Materials and Structures-I, Douglas Aircraft Company, 1954.

126. J. C. Grosskreutz, Research on the Mechanism of Fatigue, Technical Documentary Report No. WADD-TR-60-313, Part II, AF Materials Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, December 1963.

127. R. L. Segall and P. G. Partridge, Phil. Mag., 4, 912 (1959).

128. R. N. Wilson and P. J. E. Forsyth, Journal of the Institute of Metals, 87, 336 (1959).

129. K. Thomas and A. Franks, J. Appl. Phys., 30, 649 (1959).

BIBLIOGRAPHY (CONTD)

130. J. C. Grosskreutz and F. R. Rollins, J. Appl. Phys., 30, 668 (1959).

131. H. D. Avery and W. A. Backofen, Fracture of Solids, p.339, John Wiley (1963).

132. A. J. McEvily and R. C. Boettner, Acta Met., 11, 725 (1963).

133. M. L. Williams, A Review of Recent Research in GALCIT Concerning Fracture Initiation, Aeronautical Research Laboratories Report, ARL-1, January 1961.

134. G. Miller, private communications with J. G. Grosskreutz, Reference 126 above.

135. H. T. Corten and T. S. Dolan, "Cumulative Fatigue Damage," Paper No. 2 of Session 3, International Conference on Fatigue of Metals, 1, Institute of Mechanical Engineers, London (1956).

136. A. J. McEvily, Jr. and R. C. Boettner, A Note on Fatigue and Microstructure, Fracture of Solids, John Wiley and Sons, N. Y., 383-389 (1963).

137. W. A. Wood, "Recent Observations on Fatigue Failure in Metals," ASTM STP No. 237, 110-119 (1958).

138. J. Schijve, Analysis of the Fatigue Phenomenon in Aluminum Alloys, Natl. Aero- and Aeronautical Research Institute, Amsterdam, NLR-TR-M2122 (1964).

139. P. J. E. Forsyth, "A Two-Stage Process of Fatigue Crack Growth," Proc. Crack Propagation Sym., Cranfield, 76-94 (1961).

140. C. Laird and G. C. Smith, "Crack Propagation in High Stress Fatigue," Phil. Mag., 7, 847-857 (1962).

141. "The Slow Growth and Rapid Propagation of Cracks," Second Report of a Special ASTM Committee, ASTM Bulletin, May 1961, pp.389-390.

142. H. E. Romine, Critical Crack Extension Force Determination, U. S. Naval Proving Ground Technical Reports, T27-59, T3-59, T13-59, T22-58 and T8-60.

143. P. Paris, "The Growth of Cracks Due to Variation in Load," Ph.D. Dissertation, Lehigh University, Sept. 1962.

144. Fatigue Crack Growth, Workshop in Fracture Mechanics, Third Fracture Mechanics Summer Series, 7-19 August 1966, University of Denver, Denver Research Institute, Denver, Colorado.

BIBLIOGRAPHY (CONTD)

145. W. E. Anderson and P. Paris, "Evaluation of Aircraft Materials by Fracture," The Metals Engineering Quarterly, ASM Vol. 1, No. 2, May 1961.

146. J. Schijve, D. Broek, and P. de Rijk, The Effect of Frequency of an Alternating Load on the Crack Rate in a Light Alloy Sheet, NRL Report (Nat. Aero. Res. Inst., The Netherlands), TN-M2092, Sept. 1961.

147. D. R. Donaldson and W. E. Anderson, "Crack Propagation Behavior of Some Airframe Materials," Proceedings of the Crack Symposium, Cranfield, England, Sept. 1961.

148. P. Paris, "The Fracture Mechanics Approach to Fatigue," Proceedings of the 1963 Sagamore Conference, Syracuse University Press, 1964.

149. S. H. Smith, "Fatigue Crack Growth Under Uniaxial Narrow and Broadband Random Loading," Published in the Proceedings of the 2nd International Conference on Acoustic Fatigue, Syracuse University Press, 1964/1965. (Also available from the Boeing Company, Airplane Division).

150. B. Linder, private communications to P. Paris of tests results obtained at Lehigh University, Civil Engineering Department.

151. A. J. McEvily and R. C. Boettner, "On Fatigue Crack Propagation in FCC Metals," Acta Met., Vol. 11, No. 7, July 1963.

152. C. M. Carmen, J. M. Katlin, and P. Paris, "Low Cycle Fatigue of High Strength Solid Propellant Rocket Motor Materials," Proceedings of the Maraging Steel Review, Wright-Patterson AFB, June 1964.

153. J. Schijve, "Fatigue Crack Propagation in Light Alloy Sheet Material and Structures," Advances in Aeronautical Science, Vol. 3-4, Pergamon Press, 1961.

154. C. M. Hudson and H. F. Hardrath, Effect of Changing Stress Amplitude on the Rate of Fatigue Crack Propagation in Two Aluminum Alloys, NASA, TN-D-960, Sept. 1961.

155. S. H. Smith, private communications with P. Paris, The Boeing Company, Airplane Division, Renton, Washington.

156. C. D. Beacham and R. M. N. Pelloux, "Electron Fractography-A Tool for the Study of Fracture Processes," ASTM Fracture Mechanics Symposium, 67th Annual Meeting, Chicago, June 1964, ASTM STP 381.

157. J. Schijve, "Analysis of the Fatigue Phenomenon in Aluminum Alloys," Doctoral Thesis, Technische Hogeschoal Te Delft (The Netherlands), 8 July 1964.

BIBLIOGRAPHY (CONTD)

158. G. R. Irwin, Relatively Unexplored Areas of Fracture Mechanics, Dept. of T. and A. M., Report No. 240, Univ. of Ill., Feb. 1963.

159. J. Krafft, "Correlation of a Low-Medium-, and High-Strength Steel," Report to the ASTM Committee on Fracture, May 1963.

160. J. Rice, Comments on Rigid Plastic Strip Model of Fatigue Crack Growth, Unpublished Report, Lehigh University, 1963.

161. A. K. Head, "The Growth of Fatigue Cracks," Phil. Mag. Vol. 44, Series 7, p. 925, 1953.

162. N. E. Frost and D. S. Dugdale, "The Propagation of Fatigue Cracks in Sheet Specimens," Journal of Mechanics and Physics of Solids, Vol. 6, No. 2, p. 92, 1958.

163. H. W. Lui, "Crack Propagation in Thin Metal Sheets Under Repeated Loading," Journal of Basic Engineering, trans. ASME, Ser. D. Vol. 83, p. 23, 1961.

164. Lionel S. Marks, Mechanical Engineers Handbook, Fourth Edition, McGraw-Hill Book Company, Inc., New York, 1941, pp.304-307.

165. W. J. Crichlow, "The Ultimate Strength of Damaged Structure," ICAF-AGARD Symposium on Full-Scale Aircraft Fatigue Testing, Amsterdam, Netherlands, 9-11 June 1959.

166. P. H. Denke, "A General Digital Computer Analysis of Statically Intermediate Structures," AGARD paper, Aachen, Germany, September 1959.

167. P. H. Denke, "A Matrix Method of Structural Analysis," Proceedings of the Second U. S. Congress of Applied Mechanics, p.445, June 1954.

168. P. H. Denke, "The Matrix Solution of Certain Nonlinear Problems in Structural Analysis," Journal of the Aeronautical Sciences, Vol. 23, No. 3, p.231, 1956.

169. R. Hill, The Mathematical Theory of Plasticity, Oxford at The Clarendon Press, 1956.

170. G. R. Irwin, "Fracture Testing of High Strength Sheet Materials," Report of Special ASTM Committee on FTHSM, ASTM Bulletin No. 243, NRL Report 5486, July 1960.

171. P. Paris and F. Erdogan, "A Critical Analysis of Crack Propagation Laws," ASME Paper No. 62-WA-234, 1962.

BIBLIOGRAPHY (CONTD)

172. D. R. Donaldson and W. E. Anderson, "Crack Propagation Behavior of Some Air Frame Materials," Proceedings of Crack Propagation Symposium, Cranfield, 1961, p 375.

173. G. R. Irwin, Structural Mechanics, Pergamon Press, New York, 1960, p 557.

174. Che-Yu Li and R. P. Wei, "Calibrating the Electrical Potential Method for Studying Slow Crack Growth," (see Reference 176).

175. Air Force Materials Laboratory, Aerospace Structural Metals Handbook, March 1963, W-PAFB, Ohio, p 3207.

176. Che-Yu Li and R. P. Wei, "Calibrating the Electrical Potential Method for Studying Slow Crack Growth," Materials Research and Standards, August 1966, pp. 392-394.

177. R. M. Brick and Arthur Phillips, Structure and Properties of Alloys, McGraw-Hill Book Company, Inc., New York, 1949.

178. John Wulff and Howard F. Taylor, Metallurgy for Engineers, (Casting, Welding and Working), John Wiley and Sons, New York, 1952.

179. John Vaccari (Associate Editor), Wrought Aluminum and Its Alloys, Materials in Design Engineering, Reinhold Publishing Corporation, June 1965, pp. 117-132.

180. William F. Brown and John E. Srawley, American Society for Testing and Materials, National Aeronautical and Space Administration, ASTM, STP 410, Plane Strain Crack Toughness Testing of High Strength Metallic Materials, 1966, pp. 1-8.

181. H. Schardin, "Velocity Effects in Fracture," Fracture, Averbach, Feldbeck, Hahn, Thomas, eds., Wiley, New York, 297 (1959).

182. A. A. Wells and D. Post, "The Dynamic Stress Distribution Surrounding a Running Crack — A Photoelastic Analysis," Proc. SESA, 16, 69 (1958).

183. D. G. Christie, "An Investigation of Cracks and Stress Waves in Glass and Plastics by High-Speed Photography," Trans. Soc. Glass Tech., 36, 74 (1952).

184. Dennis Rader, "On the Dynamics of Crack Growth in Glass," Experimental Mechanics, April 1967, pp. 160-167.

185. D. M. Marsh, "Plastic Flow in Glass," Proc. Roy. Soc., 279, 420 (1964).

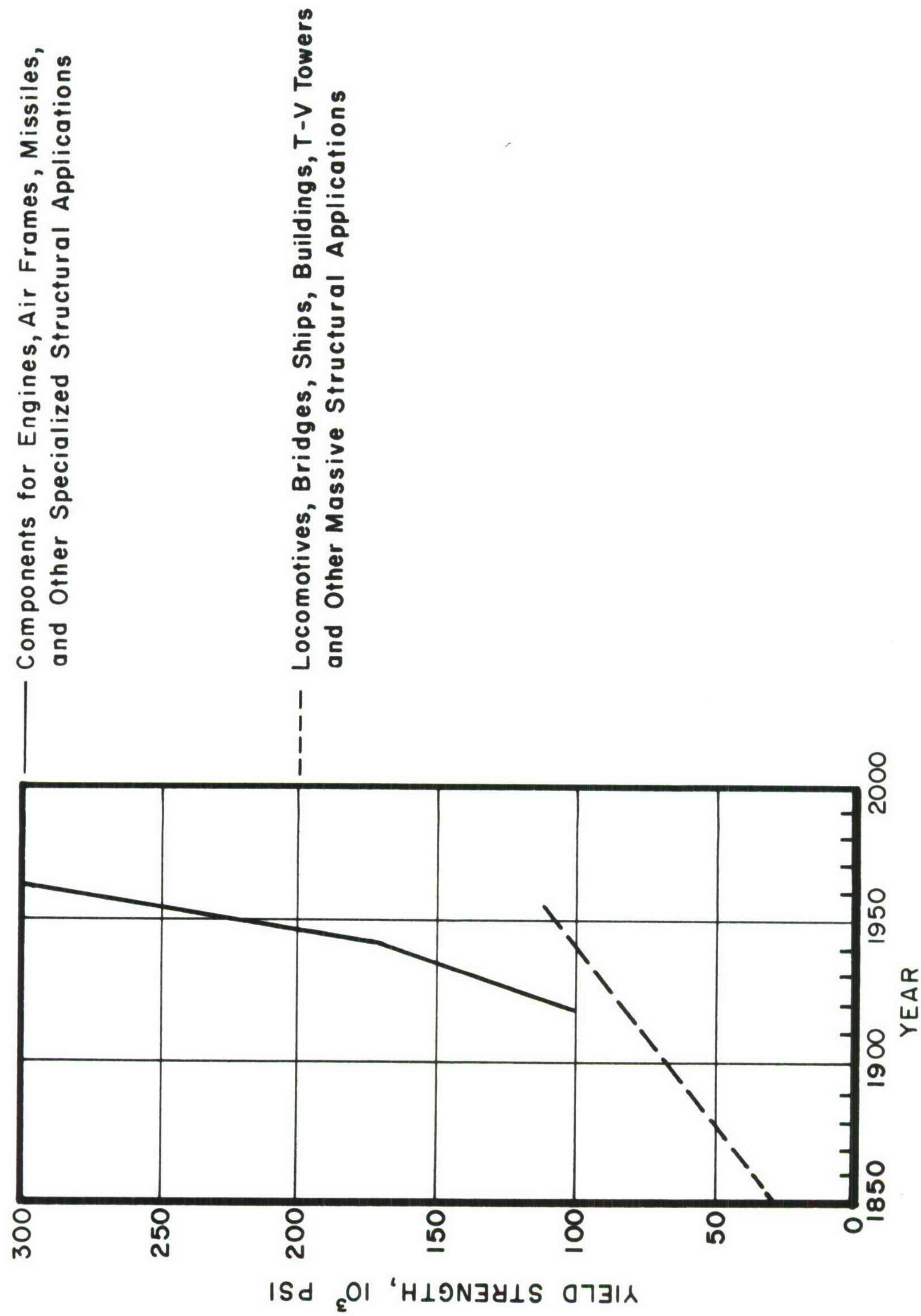
BIBLIOGRAPHY (CONTD)

186. John A. Mock, Associate Editor, "Liquid Crystals Track Flaws in a Colorfulway," Materials in Design Engineering, February 1969, pp. 66-67.

APPENDIX

1. SCHEMATIC A

2. SCHEMATIC B

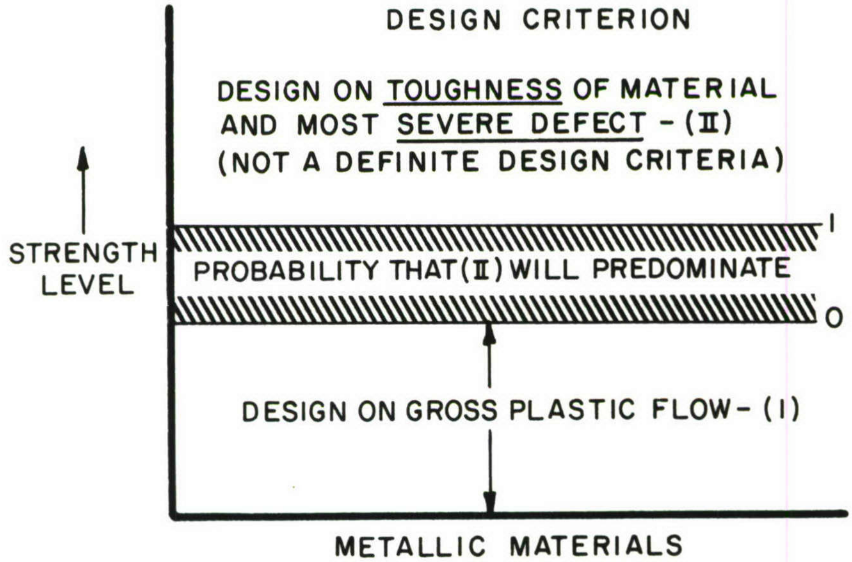


Schematic A. - Increased Yield Strength of Metallic Materials as a Function of Years

Material Characteristics

1. τ_c = critical shear stress for onset of plastic flow

2. σ_c = critical stress of normal stress at the onset of fracture



Schematic B. - Design Criterion for Two Probable Modes of Failure of Gross Plastic Flow (Tensile Instability) and Severe Defect (Brittle Fracture)

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13. ABSTRACT

The purpose of this report is to synthesize technological concepts of fracture by making a historical review of the literature from 1913 up to the present time. The pertinent aspects of fracture and the development of relevant concepts of linear elastic fracture mechanics derivatives were delineated and summarized for the prediction of the critical length of fatigue cracks. The pertinent aspects of fracture consisted of the synthesis of Ingliss, Griffith, Orowan, Irwin,and Westergaard's relevant theoretical concepts. It also delineates Boyle's analytical and experimental results of the Westergaard-Irwin theoretical compliance of through-the-thickness centrally cracked plate and sheet for the determination of plane-strain (K_{Ic}) and plane-stress (K_c) fracture toughness stress-intensity parameter of high strength alloys.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fracture						
Fatigue						
Fracture Mechanics						
Subcritical Crack Length						
Fracture Dynamics						